STUDENTS SUPPORT MATERIAL
CLASS - XII
MATHEMATICS
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तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN
AHMEDABAD REGION

STUDENT SUPPORT MATERIAL

CHIEF PATRON

DR. JAIDEEP DAS,
DEPUTY COMMISSIONER,
KVS RO AHMEDABAD

PATRON

SMT. SHRUTI BHARGAVA,
ASSISTANT COMMISSIONER,
KVS RO AHMEDABAD

SUPERVISION TEAM

MR. MOHAN CHANDRA SATYAWALI

PRINCIPAL

KV NO-1, SECTOR-30, GANDHINAGAR

CONTENT DEVELOPEMENT TEAM

SL.NO	NAME OF PGT(MATHS)	NAME OF KV	NAME OF CHAPTER
1.	SH. PRINCE KUMAR	AHMEDABAD CANTT	CH1
2.	SH. PRAVEEN KHANDELWAL	NO.1, AHMEDABAD	CH 2
3.	MRS SHILPA TANEJA	NO.1, GANDHINAGAR	CH 3
4.	SH. A. K. GUPTA	VV NAGAR	CH 4
5.	SH. L.S. RAWAT	AFS, BARODA	CH 5
6.	SH. RAJENDRA PARMAR	VALSURA	CH 6
7.	SH. A. P. SRIVASTAVA	AFS, BARODA	CH 7
8.	MRS. BHAVNA SUTARIA	CRPF, GANDHINAGAR	CH 8
9.	SH. ASHUTOSH RAI	ONGC, CHANDKHEDA	CH 9
10.	MS. SEEMA SUROLIA	AHMEDABAD CANTT	CH 10
11.	SH. SHIRIN PANDYA	KRIBHCO, SURAT	CH 11
12.	SH. PRASHANT TIWARI	INF. LINES JAMNAGAR	CH 12
13.	SH. MANISH KUMAR	NO.1, SURAT	CH 13

CH-1:- RELATION AND FUNCTION

1 MARK QUESTIONS

1. **Let A = {1,2,3} , B ={4,5,6,7} and let f ={(1,4),(2,5),(3,6)} be a function from A to B. State whether f is one- one and onto.**
2. **State the reason for the relation R in the set {1,2,3} given by R = {(1,2)(2,1)} not to be transitive.**
   Ans:- we know that ,for a relation to be transitive ,(x,y) $\in R$, (y,z) $\in R \Rightarrow$(x,z) $\in R$
   Here (1,2) $\in R$ and (2,1) $\in R$ *but* (1,1) $\notin R$. therefore R is not transitive.
3. **If R is the equivalence relation in the set A ={0,1,2,3,4,5} given by R ={(a,b): 2 divides (a-b)}, write the equivalence class of [0].**
4. **If f: R $\rightarrow$ R is the function defined by f(x) = $4x^3$ + 7 , then show that f is a surjective.**
5. **Show that function f :R $\rightarrow$R , defined by f(x) = a x + b where a , b$\in R , a \neq$0 is a bijection.**
6. **If R = {(a,a^3): a is a prime number less than 5} be a relation .find the range of R.**
7. **Let A = {1, 2, 3}. How many number of equivalence relations containing (1, 2).**
8. **Let A = {1, 2, 3} and consider the relation R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}.**
   State R is Reflexive or symmetric or transitive or equivalence relation?
9. **How many reflexive relations are possible in a set A whose n(A)=3.**
10. **Let the function *f:*R $\rightarrow$R be defined by *f* (*x*) = 4*x* − 1, $\forall x \in$R. then , show that *f* is one-one.**

11. **State whether the function f: N →N given by f(x) = 5x is injective, surjective or both.**

12. **Check whether the function f: $R \to R$ defined as (x) = x^3 is one-one or not.**

13. **Check whether the function $f: N \to N$ $defined\ as\ f(x) = x^2$ is onto or not.**

14. **Show that function $f: R \to R$ $defined\ as\ f(x) = x^4$ is not one-one.**

15. **If the set A contains 5 elements and the set B contains 6 elements, then write the number of one-one and onto mappings from A to B?**

2 MARK QUESTIONS

16. **Let Z be the set of all integers and R be the relation on Z defined as R = { (a,b): a-b is divisible by 5 ; a , b $\in Z$ }. Prove that R is the symmetric and transitive relation.**

17. **Show that the relation in the set R of real numbers, defined as S = { (a , b): a $\leq b^2$; a , b$\in R$ } is neither reflexive nor symmetric nor transitive.**

18. **Let f: X $\to Y$ be a function. Define a relation R on X given by R = {(a,b): f(a) = f(b) }.Show that R is an equivalence relation on X.**

19. **Let A=R-{3} And B = R-{1} . consider the function f :A→B defined by f(x)=$\left(\frac{x-2}{x-3}\right)$. Is one-one and onto?**

Ans:-

Let $x_1, x_2 \in A$.

Now, $f(x_1) = f(x_2) \Rightarrow \dfrac{x_1 - 2}{x_1 - 3} = \dfrac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

Hence f is one-one function.

For Onto

Let $y = \dfrac{x - 2}{x - 3}$

$\Rightarrow xy - 3y = x - 2 \Rightarrow xy - x = 3y - 2$

$\Rightarrow x(y - 1) = 3y - 2$

$\Rightarrow x = \dfrac{3y - 2}{y - 1}$ ...(i)

From above it is obvious that $\forall\, y$ except 1, *i.e.*, $\forall\, y \in B = R - \{1\}\ \exists\, x \in A$

20. Let A=R-{3} And B = R-{1} . Consider the function f :A→B defined by f(x)=$\left(\frac{x-1}{x-2}\right)$, show that f is one-one and onto.

21. Consider f : R_+ →[4,∞) given by f(x)= x^2+ 4. Show that f is a bijective function. where R_+ is the set of all non-negative real number.

<u>3 MARK QUESTIONS</u>

22. Show that the relation R in the set A = {1,2,3,4,5} given by R = {(x,y): $|x-y|$ *is divisible by* 2} is an equivalence relation.

Ans: - A = {1,2,3,4,5}

<u>Reflexive</u>:- as for any x $\in A$, we get $|x-x| = 0$, which is divisible by 2.

Therefore R is reflexive.

<u>Symmetric</u> : - as for any x $, y \in A$,

xRy⇒ we get $|x-y|$ is divisible by 2.

$|x-y| = 2\lambda$

x-y = ± 2λ

y-x =$\mp 2\lambda$

$|y-x|$**=** 2λ

$|y-x|$ **is divisible by 2.**

Therefore R is symmetric relation.

<u>Transitive</u>: - xRy$\Rightarrow$ **we get** $|x-y|$ **is divisible by 2.**

$|x-y| = 2\lambda 1$

x-y = ± $2\lambda\,1$ **-----------(1)**

yRz$\Rightarrow$ **we get** $|y-z|$ **is divisible by 2.**

$|y-z| = 2\lambda 2$

y-z = ± $2\lambda 2$ **-----------(2)**

adding equation (1) and (2)

x-y + y-z =± $2\lambda 1$ $\pm\ 2\lambda 2$

x-z = ± $2(\lambda 1 + \lambda 2)$ **= ±** 2λ

$|x-z| = 2\lambda$

xRz

therefore R is a transitive relation.

23. Let A = {1,2,3…………9} and R be the relation in A×A defined by (a,b)R (c,d) if a+d=b+c for (a,b),(c,d) in A×A. prove that R is an equivalence relation.

24. Show that the function f in A =R$-\left\{\frac{2}{3}\right\}$ **defined as f(x) =**$\frac{4x+3}{6x-4}$ **is one-one and onto.**

25. Consider f: R- $\left\{\frac{-4}{3}\right\}$ **→ R -**$\left\{\frac{4}{3}\right\}$ **given by f(x) =** $\frac{4x+3}{3x+4}$ **, show that f is bijective function.**

26. Show that the function f in A = R – {$-\frac{4}{3}$**} defined as**$f(x) = \frac{4x}{3x+4}$ **is one – one. Also check whether f is onto or not.**

<u>5 MARK QUESTIONS</u>

27. Consider f:R_+ **→[-9,∞) given by f(x)= 5**x^2**+6x-9. Show that f is is bijective function. where** R_+ **is the set of all non-negative real number.**

28. Let f :N→R be a function defined as f(x)=$4x^2+12x+15$ .Show that F:N→S, Where S is range of f , is is bijective function.

29. Consider f:R_+ →[-5,∞) given by f(x)= $9x^2+6x-5$. Show that f is bijective function. where R_+ is the set of all non-negative real number.

30. Shoe that f :N→N ,given by f(x)=$\begin{Bmatrix} x+1 & when\ n\ is\ odd \\ x-1 & when\ n\ is\ even \end{Bmatrix}$ is both one-one and onto.

Ans: -

For one-one

Case I : When x_1, x_2 are odd natural number.

$\therefore \quad f(x_1) = f(x_2) \Rightarrow x_1+1 = x_2+1 \quad \forall\, x_1, x_2 \in N$

$\Rightarrow x_1 = x_2$

i.e., *f* is one-one.

Case II : When x_1, x_2 are even natural number

$\therefore \quad f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

i.e., *f* is one-one.

Case III : When x_1 is odd and x_2 is even natural number

$f(x_1) = f(x_2) \Rightarrow x_1+1 = x_2-1$

$\Rightarrow x_2 - x_1 = 2$ which is never possible as the difference of odd and even number is always odd number.

Hence in this case $f(x_1) \neq f(x_2)$

i.e., *f* is one-one.

Case IV: When x_1 is even and x_2 is odd natural number

Similar as case III, We can prove *f* is one-one

For onto:

$\therefore \quad f(x) = x+1$ if x is odd

$= x-1$ if x is even

$\Rightarrow$ For every even number 'y' of codomain $\exists$ odd number $y-1$ in domain and for every odd number y of codomain $\exists$ even number $y+1$ in Domain.

i.e. *f* is onto function.

Hence *f* is one-one onto function.

31. Let f :N→N be defined by f(n)=$\begin{Bmatrix} \frac{n+1}{2} & when\ n\ is\ odd \\ \frac{n}{2} & when\ n\ is\ even \end{Bmatrix}$ for all n ∈ N.state wheter the function f is bijective .Justify your answer.

CASE STUDY QUESTION

In two different societies, there are some school going students - including girls as well as boys. Satish forms two sets with these students, as his college project. Let A =

$\{a_1, a_2, a_3, a_4, a_5\}$ **and B =** $\{b_1, b_2, b_3, b_4,\}$ **where** a_i **'s and** b_i **'s are the school going students of first and second society respectively. Satish decides to explore these sets for various types of relations and functions.**

Using the information given above, answer the following

(i) Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible?

(a) 0

(b) 2^5

(c) 2^{10}

(d) 2^{20}

(ii) let R: A → B ,R ={ (x, y) : x and y are students of same sex}.then relation R is

(a) reflexive only

(b) reflexive and symmetric but not transitive

(c) reflexive and transitive but not symmetric

(d) an equivalence relation

(iii) Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B, separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find the symmetric relation on set B. What is difference between their results?

(a) 1024

(b) $2^{10}(15)$

(c) $2^{10}(31)$

(d) $2^{10}(63)$

(iv) Let R: A → B , R = $\{(a_1, b_1), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$ **then R is**

(a) neither one-one nor onto

(b) one-one but, not onto

(c) only onto, but not one-one

(d) one-one and onto both

(V) To help Satish in his project, Rajat decides to form onto function from set A to B. How many such functions are possible?

(a) 342

(b) 243

(c) 729

(d) 1024

BY PRINCE KUMAR PGT(MATHS)-KV AHMEDABAD CANTT

CH: 2 INVERSE TRIGONOMETRIC FUNCTIONS

1 MARK QUESTIONS

Q1. If $\sin^{-1}x > \cos^{-1} x$ then x belongs to which interval?

Ans: $(\frac{1}{\sqrt{2}}, 1)$

Q2.$\cos^{-1}(\cos x) = x$ then value of x is………

Ans: $x \in [0,\pi]$

Q3. The domain of the function defined by

$f(x) = \sin^{-1} \sqrt{x-1}$

Soln: Here for domain of $\sin^{-1} \sqrt{x-1}$

But for domain : $0 \le x - 1 \le 1$, therefore $1 \le x \le 2$;

hence x ε [1 ,2]

Q4. Find value of $\cos\{\sin^{-1}(-\frac{3}{5})\}$

Soln: $\sqrt{1^2 - (-\frac{3}{5})^2} = \frac{4}{5}$

Q5. Find the value of

tan-1(1) + cos-1(-1/2) + sin -1 (-1/2)

Soln: $\frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{9\pi}{8}$

Q6.What is the principal value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$ *?*

***Soln:* $\pi - \frac{5\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$**

Q7. Write The domain of the function $\sin^{-1}2x + \cos^{-1}2x$

Ans: $[-\frac{1}{2}, \frac{1}{2}]$

Q8.Write the range of the function sin ($\sin^{-1}x + \cos^{-1}x$)

Ans: {1}

Q9.If cos($\tan^{-1}x + \cot^{-1}\sqrt{3}$) = 0, then find value of x ?

Ans: $x = \sqrt{3}$

Q10.For what value of x , $\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}$ is true ?

Ans: R – (-1,1)

2 MARKS QUESTIONS

Q1. Find the range of f(x) = $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x$

Soln: Here, D_f = [-1 ,1]

Therefore, range of f(x): [f (-1), f (1)]

$=[-\frac{\pi}{2} + \pi - \frac{\pi}{4}, \frac{\pi}{2} + \pi - \frac{\pi}{4}] = [\frac{\pi}{4}, \frac{3\pi}{4}]$

Q2. If $\tan^{-1}(\cot\theta) = 2\theta$, then find θ

Soln: since $\tan^{-1}(\cot\theta) = \tan^{-1}\{\tan(\frac{\pi}{2} - \theta)\} = 2\theta$

Therefore $(\frac{\pi}{2} - \theta) = 2\theta$

or $\frac{\pi}{2} = 3\theta$; hence $\theta = \frac{\pi}{6}$

Q3. Prove that $\sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3) = 15$.

Soln: LHS

$1 + \{\tan(\tan^{-1} 2)\}^2 + 1 + \{\cot(\cot^{-1} 3)\}^2$

=1+4+1+9 =15 = RHS

Q4.if 4$\cos^{-1}x$ + $\sin^{-1}x$ = π, find value of x

Soln:3$\cos^{-1}x$ + $\cos^{-1}x$ + $\sin^{-1}x$ = π

3$\cos^{-1}x$ + $\frac{\pi}{2}$ = π

3$\cos^{-1}x$ = $\frac{\pi}{2}$

x = $cos\frac{\pi}{6}$ = $\frac{\sqrt{3}}{2}$

Q5. find value of

$\tan^{-1}(\frac{a-b}{1+ab})$ + $\tan^{-1}(\frac{b-c}{1+bc})$+ $\tan^{-1}(\frac{c-a}{1+ca})$;where a,b,c>0

soln:

$\tan^{-1} a - \tan^{-1} b + \tan^{-1} b - \tan^{-1} c + \tan^{-1} c - \tan^{-1} a = 0$

<u>3 MARKS QUESTIONS</u>

Q1.Prove that if $\sin^{-1}(2x\sqrt{1-x^2})$ = 2 $\cos^{-1}$ x then value of x ε $[\frac{1}{\sqrt{2}},1]$

Soln: Put $\cos^{-1}$ x = y, so that x = cos y
Then , 0 ≤ y ≤ π and |x|≤1-------(1) $\sin^{-1}$ (2cosysiny)= $\sin^{-1}$ (sin2y)=2y

therefore 2y ε [$-\frac{\pi}{2}, \frac{\pi}{2}$]------(2)

by equation 1 and 2

y ε [0, $\frac{\pi}{4}$] i.e. $\cos^{-1}$ x ε [0, $\frac{\pi}{4}$]

therefore x ε $[\frac{1}{\sqrt{2}},1]$

Q2. Prove that $\cot^{-1}\{\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\}$ = $\frac{x}{2}$:

where 0$\leq x \leq \frac{\pi}{4}$

Soln:

$Since\ \sqrt{1+\sin x} = \sqrt{2}\cos(\frac{\pi}{2} - \frac{x}{2})\ and\ \sqrt{1-\sin x} = \sqrt{2}\sin(\frac{\pi}{2} - \frac{x}{2})$

Hence $Cot^{-1}\{\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\} = Cot^{-1}\{\tan(\frac{\pi}{2} - \frac{x}{2})\}$

$= Cot^{-1}\{\cot\{\frac{\pi}{2} - (\frac{\pi}{2} - \frac{x}{2})\} = \frac{x}{2}$

Q3. Simplify : $\cos^{-1}(\frac{3}{5}\cos x + \frac{4}{5}\sin x)$

Soln: let $\frac{3}{5}$ **= sin y hence cos y =** $\frac{4}{5}$

Using above: $\cos^{-1}(\frac{3}{5}\cos x + \frac{4}{5}\sin x)$

$= \cos^{-1}(\sin y \cos x + \cos y \sin x)$

$= \cos^{-1}\{\sin(x+y)\}$

$= \cos^{-1}[\cos\{\frac{\pi}{2} - (x+y)\}]$

$= \frac{\pi}{2} - (x+y)\} = \frac{\pi}{2} - x - y = \frac{\pi}{2} - x - \sin^{-1}(\frac{3}{5})$

$= \{\frac{\pi}{2} - \sin^{-1}(\frac{3}{5})\} - x = \cos^{-1}(\frac{3}{5}) - x$

Q4. Express $\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$ **in simplest form.**

Soln: let $\sin^{-1}(x) = \alpha$, $\sqrt{1-x^2} = \cos\alpha$

& $\sin^{-1}(\sqrt{x}) = \beta$, $\sqrt{1-x} = \cos\beta$

Then $\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$

$= \sin^{-1}(\sin\alpha\cos\beta - \sin\beta\cos\alpha)$

$= \sin^{-1}\{\sin(\alpha - \beta)\} = \alpha - \beta = \sin^{-1}(x) - \sin^{-1}(\sqrt{x})$

Q5. Find value of

Cot [tan-^{1} x + tan^{-1}($\frac{1}{x}$)] + cos^{-1}(x) + cos^{-1}(-x); x>0

Soln: Cot [tan-^{1} x + cot^{-1}(x)] + cos^{-1}(x) + cos^{-1}(-x)

= Cot ($\frac{\pi}{2}$) + cos^{-1}(x) + π - cos^{-1}(x)

=0 + π = π

Q6. Find greatest and least value of (cos^{-1}x) ^{2}+ (sin^{-1}x)^2

Soln: As (cos^{-1}x) ^{2}+ (sin^{-1}x)^2 = ($\frac{\pi}{2}$ − sin-^{1}x) ^{2}+ (sin^{-1}x)^2

= 2[(sin^{-1}x - $\frac{\pi}{4}$)^2 +($\frac{\pi}{4}$)^2]

Therefore greatest value is 2($\frac{\pi}{4}$)^2 and least value is 2[($-\frac{\pi}{2} - \frac{\pi}{4}$)^2 +($\frac{\pi}{4}$)^2] = 5$\frac{\pi^2}{4}$

Q7. If a_1, a_2, a_3……………………, a_n is an arithmetic progression with common difference d, then evaluate the following expression.

tan{tan^{-1}($\frac{d}{1+a_1 a_2}$) + tan^{-1}($\frac{d}{1+a_2 a_3}$)+ tan^{-1}($\frac{d}{1+a_3 a_4}$) +……………… tan^{-1}($\frac{d}{1+a_n a_{n-1}}$)}

Soln: Put d = a_2 - a_1 = a_3 – a_2 = a_4 – a_3 =…………….= a_n – a_{n-1}

tan{tan^{-1}($\frac{d}{1+a_1 a_2}$) + tan^{-1}($\frac{d}{1+a_2 a_3}$)+ tan^{-1}($\frac{d}{1+a_3 a_4}$) +……………… tan^{-1}($\frac{d}{1+a_n a_{n-1}}$)}

= tan [tan-^{1} a_2 - tan-^{1} a_1 + tan-^{1} a_3 - tan-^{1} a_2 + tan-^{1} a_4 - tan-^{1} a_3 +………………………………….+ tan-^{1} a_n - tan-^{1} a_{n-1}

= tan [tan-^{1} a_n - tan-^{1} a_1] = $\frac{a_n - a_1}{1+a_n a_1}$

BY Sh. Praveen Khandelwal,PGT(MATHS) BY KV No.1, Ahmedabad

CH - 3 : MATRICES

1 MARK QUESTIONS:

Q 1. Give an example of a row matrix which is also a column matrix.

Q 2. If AB = A and BA = B then A^2 will be equal to what.

Q 3. Give an example of two matrices A and B such that A ≠ O , B ≠ O, AB = O and BA ≠ O

Q 4. Give an example of three matrices A , B and C such that AB = AC but B ≠ C.

Q 5. If A = [3   5] , B = [7 3] ,then find a non zero column matrix such that AC = BC

Q 6. If A is square matrix such that $A^2 = A$ Find the value of $(I + A)^3$

2 MARK QUESTIONS:

Q 7. Find non zero value of x , satisfying equation :

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$$

Q 8 . Solve the matrix equation :

$$\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

Q 9. The monthly incomes of Alhaz and Amir are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7 .If each saves ₹ 15000 per month , find their monthly income using matrix method.

Q 10.In a certain city , there are 30 colleges .Each college has 15 teachers , 6 clerks , 1 typist and 1 section officer. Express the given information as a column matrix. Using scalar multiplication, find the total number of posts of each kind in all the colleges.

Q 11. If A = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, B = $\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and (A + B)^2 = A ^{2} + B^2 Find a and b

Q 12. Find x such that [1 x 1] $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}$ = O

3 MARK QUESTIONS:

Q 13. If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ A = $\begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$ Find A

Q 14. Show that a matrix which is both symmetric and skew symmetric is null matrix.

Q 15. Show that the elements on the main diagonal of a skew symmetric matrix are all zero.

Q 16 . If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying A A^T = 9 I_3 , then find the value of a and b

5 MARK QUESTION :

Q 17 . Let A = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and f (x) = x ^{2} - 4 x + 7 Show that f (A) = O
Use this result to find A ^{5}

Q 18. Three schools A , B and C organised a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand made fans , mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹50 each .The number of articles sold are given below :

Article \ School	A	B	C
Hand – fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also, find the total funds collected for the purpose.

CASE STUDY BASE QUESTION

Two farmers Ram Kishan and Gurcharan Singh cultivate only three varities of rice namely X , Y and Z . The sale (in ₹) of these varities of rice by both the farmers in the month of September and October are given by the following matrices A and B

September sales (in ₹)

$$A = \begin{array}{c} \\ \\ \end{array}\begin{matrix} X & Y & Z \\ \end{matrix}$$

$$A = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} \begin{matrix} \mathit{RAM\ KISHAN} \\ \mathit{GURCHARAN\ SINGH} \end{matrix}$$

October sales (in ₹)

$$\begin{matrix} X & Y & Z \end{matrix}$$

$$B = \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \begin{matrix} \mathit{RAMKISHAN} \\ \mathit{GURCHARAN\ SINGH} \end{matrix}$$

Answer the following questions :

(a) What were the combined sales in September and October for each farmer in each variety.
(b) What was the change in sales from September to October?
(c) If Ram Kishan receive 2 percent profit on gross rupees sales, compute the profit of Ram Kishan for each variety sold in October.
(d) If Gurcharan receive 3 percent profit on gross rupees sales , compute the profit of Gurcharan Singh for each variety sold in October.

(e) Who receive more profit.

Solution:

1. [4] or [10] or any other example
2. AB = A and BA = B
   (AB) A = A A
   A(BA) = A^2

AB = A^2 (As BA = B)

A = A^2 (AB = A)

3. If A = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ and B = $\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix}$, then A ≠ O and B ≠ O

But AB = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ = O

BA = $\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\3 & 0\end{bmatrix} \neq$ O

4. Let A = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$, B = $\begin{bmatrix}0 & 0\\-1 & 0\end{bmatrix}$ and C = $\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$

AB = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\-1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ AC = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ But $B \neq C$

5.

Let C = $\begin{bmatrix}a\\b\end{bmatrix}$ Then AC = BC

gives 3 a + 5 b = 7 a + 3 b

4 a = 2 b

2 a = b

C = $\begin{bmatrix}a\\2a\end{bmatrix}$ where a belongs to R

6. $(I + A)^3 = I^3 + A^3 + 3 I^2 A + 3 A^2 I$

$= I + A^2 A + 3 A + 3 A^2$

$= I + A A + 3 A + 3 A$

$= I + A^2 + 6 A$

$= I + A + 6 A$

$= I + 7 A$

7. x $\begin{bmatrix}2x & 2\\3 & x\end{bmatrix}$ + 2 $\begin{bmatrix}8 & 5x\\4 & 4x\end{bmatrix}$ = 2 $\begin{bmatrix}x^2+8 & 24\\10 & 6x\end{bmatrix}$

$\begin{bmatrix}2x^2 & 2x\\3x & x^2\end{bmatrix} + \begin{bmatrix}16 & 10x\\8 & 8x\end{bmatrix} = \begin{bmatrix}2x^2+16 & 48\\20 & 12x\end{bmatrix}$

$\begin{bmatrix}2x^2+16 & 2x+10x\\3x+8 & x^2+8x\end{bmatrix} = \begin{bmatrix}2x^2+16 & 48\\20 & 12x\end{bmatrix}$

2 x + 10 x = 48 , 3 x + 8 = 20 , $x^2 + 8x$ = 12 x

12 x = 48 , 3 x = 12 , x^2 = 4 x

X = 4 , x = 4 , x = 0 , 4

X = 4 is non zero solution.

8. $\begin{pmatrix} x^2 - 3x \\ y^2 - 6y \end{pmatrix}$ = $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$ gives

$x^2 - 3x$ = -2 i.e. $x^2 - 3x$ + 2 = 0 and

$y^2 - 6y$ = 9 i.e. $y^2 - 6y$ - 9 = 0

$on\ solving\ x = 1, 2\ and\ y = 3 \pm 3\sqrt{2}$

9. $\begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15{,}000 \\ 15{,}000 \end{bmatrix}$

3 x – 5 y = 15000

And 4 x – 7 y = 15000

Gives x = 30,000 and y = 15,000

Alhaz monthly income 3 x 30,000 = Rs 90,000

Amir monthly income 5 x 30,000 = Rs 1,50,000

10. A is a column matrix containing 15 , 6, 1, 1and total number of post in 30 colleges 30 A represented by 450 , 180, 30 , 30

11. (A + B) (A+B) = A ^{2} + B^2

A ^{2} + AB + BA + B^2 = A ^{2} + B^2

Gives BA + AB = 0

$$\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} + \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2a-b+2 & -a+1 \\ 2a-2 & -b+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2 a – b + 2= 0 , -a + 1=0 , 2 a -2 = 0 and –b+4 = 0

a = 1 and b = 4

12. [1 x 1] $\begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix}$ = O

7 + 2 x + 1 2 x + x ^{2} + 21 + 2 x = 0

X ^{2} + 16 x + 28 = 0

(x + 14) (x + 2) = 0

X = - 14 , x = -2

13. Let A = $\begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}$

In LHS on multiplication of two matrix we get

$$\begin{bmatrix} 2x-a & 2y-b & 2z-c \\ x & y & z \\ -3x+4a & -3y+4b & -3z+4c \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

2 x – a = -1 , x = 1 , 3 x + 4 a = 9 , 2 y – b = -8 , y = -2

-3 y + 4 b = 22 , 2 z – c = -10 , z = -5 and c = 0

X = 1 , a = 3 , y = -2 , b = 4 , z = - 5 and c = 0

A = $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

14. Let A = [a_{ij}]
If A = [a_{ij}] is symmetric matrix then
$a_{ij} = a_{ji}$ for all i , j -------(i)
Also A = [a_{ij}] is skew symmetric then
$a_{ij} = - a_{ji}$ for all i , j -------(ii)
from (i) and (ii)

$a_{ij} = - a_{ij}$ for all i , j
2 a_{ij} = 0 for all i, j
a_{ij} = 0 for all i , j
A = [a_{ij}] is a null matrix

15. A = [a_{ij}] is skew symmetric then
$a_{ij} = - a_{ji}$ for all i , j
$a_{ii} = - a_{ii}$ for all values of i
2 a_{ii} = 0
a_{ii} = 0 for all values of i
$a_{11} = a_{22} = a_{33} =$ ________ $= a_{nn} = 0$

16. A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ A^T = $\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$ $A A^T = 9 I_3$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 0 & a+2b+4 \\ 0 & 9 & 2a+2-2b \\ a+2b+4 & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

a+ 2 b + 4 = 0 , 2 a + 2 – 2 b = 0 and $a^2 + 4 + b^2 = 9$

we get a = - 2 and b = -1

17. f (A) = $A^2 - 4A + 7 I_2$

$A^2 = \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$

$- \ 4A = \begin{bmatrix} -8 & 12 \\ 4 & -8 \end{bmatrix} \quad 7 \ I_2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$f(A) = A^2 - 4A + 7I_2$

$f(A) = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} -8 & 12 \\ 4 & -8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$f(A) = \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Now f (A) = O

$A^2 - 4A + 7I_2 = 0$

$A^2 = 4A - 7I_2$

$A^3 = A^2 A = (4A - 7I_2)A = 4A^2 - 7I_2 A$

Put the value of A^2

$A^3 = 4(4A - 7I_2) - 7A$

$A^3 = 9A - 28I_2$

Similarly $A^4 = A^3 A$

$= (9A - 28I_2)A$

$= 9A^2 - 28A$

$= 9(4A - 7I_2) - 28A$

$= 8A - 63I_2$

In the same way

$A^5 = -31A - 56I_2$

$A^5 = -31 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

18. Three items sold by three school are represented by the following 3 x 3 matrix Q as given below.

Hand –fans Mats Plates

$$Q = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}$$

The price matrix representing price of three articles in ₹ is a 3 x 1 matrix given by

$$P = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

The funds collected by schools A , B and C separately by selling three articles are given by the product matrix

$$QP = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$QP = \begin{bmatrix} 40\,X\,25 & +\,50\,X\,100 & +\,20\,X\,50 \\ 25\,X\,25 & +40\,X\,100 & +\,30\,X\,50 \\ 35X25 & +\,50\,X\,100 & +\,40\,X\,50 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

Hence the funds collected by schools A , B and C are ₹ 7,000 , ₹ 6125 and ₹ 7875 respectively .The total funds collected = ₹(7100+ 6125+ + 7875) = ₹21,000

CASE STUDY BASED QUESTION

(a)

$$A + B = \begin{bmatrix} 15{,}000 & 30{,}000 & 36{,}000 \\ 70{,}000 & 40{,}000 & 20{,}000 \end{bmatrix} \begin{matrix} RAMKISHAN \\ GURCHARAN\ SINGH \end{matrix}$$

(b)

A − B = $\begin{bmatrix} 5{,}000 & 10{,}000 & 24{,}000 \\ 30{,}000 & 20{,}000 & 0 \end{bmatrix}$ $\begin{matrix} RAM\ KISHAN \\ GURCHARAN\ SINGH \end{matrix}$

(c)

2 % of B first row = $[\ 100 \quad 200 \quad 120\ ]$

(d)

3 % of B second row = $[\ 600 \quad 300 \quad 300\ ]$

(e)

Gurcharan Singh receive more profit

BY MRS. SHILPA TANEJA, PGT(MATHS) KV NO.1, GANDHINAGAR

CH- 4 : DETERMINANTS

ONE MARK QUESTIONS

1. If A is a square matrix of order 3 and $|A| = 3$, then find $|A.adjA|^3$.

   Solution: Since $|A.adjA| = |A|^n$

   $so\ |A.adjA| = 3^3 = 27$

   $hence\ |A.adjA|^3 = 27^3 = 19683$ **Ans.**

2. Find the value of $\begin{vmatrix} 2021 & 2020 \\ 2019 & 2018 \end{vmatrix}$.

   Solution: $\begin{vmatrix} 2021 & 2020 \\ 2019 & 2018 \end{vmatrix}$

   $= \begin{vmatrix} 2021 - 2020 & 2020 \\ 2019 - 2018 & 2018 \end{vmatrix}$ $Apply\ C_2 \to C_2 - C_1$

   $= \begin{vmatrix} 1 & 2020 \\ 1 & 2018 \end{vmatrix} = 2018 - 2020 = -2$

   or $\begin{vmatrix} 2021 & 2020 \\ 2019 & 2018 \end{vmatrix} = 2021 \times 2018 - 2019 \times 2020 = 4078378 - 4078380 = -2$ **Ans.**

3. If A is any square matrix and $|A| = 5$, find $|A^n|$ for positive integer n.

   Solution: Since $|A^n| = |A|^n\ so\ |A^n| = 5^n$

4. For what value of $'k'$in $A = \begin{bmatrix} 2 & k & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, A^{-1} exist.

   Solution: A^{-1} exist if $|A| \neq 0$

   $or \begin{vmatrix} 2 & k & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix} \neq 0$

   $or\ 2(6-5) - k(0-5) - 3(0-2) \neq 0$

   $or\ 2 + 5k + 6 \neq 0$

   $or\ 5k \neq -8\ or\ k \neq -\frac{8}{5}$ **Ans.**

5. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, find $|adjA|$

   Solution: $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ $so\ |A| = \begin{vmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = a^3$

Since $|adjA| = |A|^{n-1} = (a^3)^{3-1} = a^6$ **Ans.**

6. If A is a square matrix and $A^2 = A$, find $|A|$

   Solution: Since $A^2 = A \; so \; |A|^2 = |A|$

   $so \; |A|^2 - |A| = 0$

   $so \; |A|(|A| - 1) = 0$

   $so \; |A| = 0 \; and \; |A| - 1 = 0$

   $so \; |A| = 0 \; and \; |A| = 1$ **Ans.**

7. Any square matrix A is called orthogonal matrix if $AA' = I$. Find $|A|$.

   Solution: $since \; AA' = I$

   $so \; |AA'| = |I|$

   $so \; |A||A'| = 1$

   $so \; |A||A| = 1$

   $so \; |A|^2 = 1 \; so \; |A| = \pm 1$ **Ans.**

   Write true/false. Justify your answer.

8. Let $A = \begin{bmatrix} 1 & \sin x & 1 \\ -\sin x & 1 & \sin x \\ -1 & -\sin x & 1 \end{bmatrix} where \; o \leq x \leq 2\pi, then \det A \in (2, \infty)$

   Ans. False.

   Justification: $|A| = \begin{vmatrix} 1 & \sin x & 1 \\ -\sin x & 1 & \sin x \\ -1 & -\sin x & 1 \end{vmatrix} Expand \; along \; R_1$

   $|A| = 1(1 + \sin^2 x) - \sin x(-\sin x + \sin x) + 1(\sin^2 x + 1)$

   $|A| = 2(1 + \sin^2 x)$

   $since - 1 \leq \sin x \leq 1$

   $so \; 0 \leq \sin^2 x \leq 1$

   $so \; 1 \leq 1 + \sin^2 x \leq 2$

   $so \; 2 \leq 2(1 + \sin^2 x) \leq 4$

   $so \; |A| \in [2, 4]$

9. If A is any invertible matrix of order 3, then show that $\det A^{-1} = \frac{1}{\det A}$

   Ans: True

Justification: We know that $A.A^{-1} = I$

$so\ |AA^{-1}| = |I|$

$so\ |A||A^{-1}| = 1$

$so\ |A^{-1}| = \frac{1}{|A|}\ or\ detA^{-1} = \frac{1}{\det A}$

10. If A is any invertible matrix then $adjA = |A|A^{-1}$

Ans: True

Justification: since $(adjA)A = |A|I,$ so post multiply by $A^{-1},$ we get

$(adjA)A.A^{-1} = |A|IA^{-1}$

$or\ (adjA)I = |A|A^{-1}$

$or\ (adjA) = |A|A^{-1}$

<u>Two Marks Questions</u>

1. If $\begin{vmatrix} \sin x & \cos x \\ \sin x & \sin x \end{vmatrix} = \begin{vmatrix} \cos x & \cos x \\ \cos x & \sin x \end{vmatrix}, x \in \left(0, \frac{\pi}{2}\right), find\ x.$

Solution: since $\begin{vmatrix} \sin x & \cos x \\ \sin x & \sin x \end{vmatrix} = \begin{vmatrix} \cos x & \cos x \\ \cos x & \sin x \end{vmatrix}$

So $\sin^2 x - \sin x \cos x = \sin x \cos x - \cos^2 x$

So $\sin^2 x + \cos^2 x = \sin x \cos x + \sin x \cos x$

So $1 = 2 \sin x \cos x$

$or\ 1 = \sin 2x$

$or \sin 2x = \sin \frac{\pi}{2}$

$or\ 2x = \frac{\pi}{2}\ or\ x = \frac{\pi}{4}$. **Ans.**

2. For any non-zero square matrix A of order 3, show that $|adjA| = |A|^2$

Solution: Since $(adjA)A = |A|I$

So $(adjA)A = |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$(adjA)A = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$

$Writing\ determinant\ of\ both\ sides$

$|(adjA)A| = \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix}$

$|(adjA)|.|A| = |A|^3 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = |A|^3$

$or\ \ |adjA| = |A|^2$ **HP**

3. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ $and\ B = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ $find\ (AB)^{-1}$

Solution: $AB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1+2 & 2-2 \\ 2+1 & -4-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & -5 \end{bmatrix}$

$|AB| = -5 - 0 = -5$

$adjA = \begin{bmatrix} -5 & 0 \\ -3 & 1 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{|AB|}(adjA) = \frac{1}{-5}\begin{bmatrix} -5 & 0 \\ -3 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 5 & 0 \\ 3 & -1 \end{bmatrix}$ **Ans.**

4. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ $and\ \Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ $then\ show\ that\ \frac{d}{dx}(\Delta_1) = 3\Delta_2$

Solution: $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$

$so\ \frac{d}{dx}(\Delta_1) = \frac{d}{dx}\left\{\begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}\right\}$

$\frac{d}{dx}(\Delta_1) = \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$

$\frac{d}{dx}(\Delta_1) = \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\begin{vmatrix} x & b \\ a & x \end{vmatrix}$

So $\frac{d}{dx}(\Delta_1) = 3(\Delta_2)$ **Verified.**

5. There are two values of 'a' for which $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then find difference of their squares.

Solution: $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, expand along C_1

$1(2a^2+4)-2(-4a-20)+0=86$

$2a^2+4+8a+40-86=0$

$2a^2+8a-42=0$

$a^2+4a-21=0$

$or\ (a+7)(a-3)=0$

$or\ a=-7\ and\ a=3$

Difference of squares= $(-7)^2-3^2=49-9=40$.**Ans.**

6. The given system of equations $x - ky - z = 0,\ kx - y - z = 0,\ x + y - z = 0$ has non-zero solution then find the possible value(s) of k.

Solution: since the given system has non-zero solution so

$\begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0\ Expand\ along\ R_1$

$or\ 1(1+1)+k(-k+1)-1(k+1)=0$

$or\ 2-k^2+k-k-1=0$

$or\ 1=k^2$

$or\ k=\pm 1$. **Ans**

<u>Three Marks Questions</u>

1. If $-1 \le x < 0, 0 \le y < 1, 1 \le z < 2\ and\ [\ \ ]$ denotes greatest integer function then choose the correct solution for $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix} =$

$[x]\ or\ [y] or\ [z]$

Solution: $since\ -1 \le x < 0, 0 \le y < 1, 1 \le z < 2$

$so -1 \le x < 1\ gives\ [x]=-1,$

$0 \ge\ y < 1\ gives\ [y]=0,$

$1 \leq z < 2\ gives\ [z] = 1$

$$so\begin{vmatrix}[x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1\end{vmatrix}$$

$$= \begin{vmatrix}-1+1 & 0 & 1 \\ -1 & 0+1 & 1 \\ -1 & 0 & 1+1\end{vmatrix}$$

$$= \begin{vmatrix}0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2\end{vmatrix}$$

$= 0 - 0 + 1(0 + 1)$

$= 1 = [z]$ **Ans.**

2. If $f(x) = \begin{vmatrix}\cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x\end{vmatrix}, then\ find\ \lim_{x\to 0}\frac{f(x)}{x^2}$

Solution: $f(x) = \begin{vmatrix}\cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x\end{vmatrix}$, Expand along C_1

$f(x) = \cos x\,(x^2 - 2x^2) - 2\sin x\,(x^2 - x) + \sin x\,(2x^2 - x)$

$f(x) = -x^2\cos x - 2x^2\sin x + 2x\sin x + 2x^2\sin x - x\sin x$

$f(x) = -x^2\cos x + x\sin x$

$$f(x) = x^2\left(-\cos x + \frac{\sin x}{x}\right)$$

$$\frac{f(x)}{x^2} = \left(-\cos x + \frac{\sin x}{x}\right)$$

$$\lim_{x\to 0}\frac{f(x)}{x^2} = \lim_{x\to 0}\left(-\cos x + \frac{\sin x}{x}\right)$$

$$\lim_{x\to 0}\frac{f(x)}{x^2} = -\cos 0 + 1$$

$\lim_{x\to 0}\frac{f(x)}{x^2} = -1 + 1 = 0$ **Ans.**

3. If $A = \begin{bmatrix}0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1\end{bmatrix}\ and\ A^{-1} = \begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{bmatrix}, then\ find\ a\ and\ c.$

Solution: $since\ AA^{-1} = I$

$$so\begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\3 & a & 1\end{bmatrix}\begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\-4 & 3 & c\\\frac{5}{2} & -\frac{3}{2} & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$so\,\frac{1}{2}\begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\3 & a & 1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\-8 & 6 & 2c\\5 & -3 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$so\,\frac{1}{2}\begin{bmatrix}2 & 0 & 2c+2\\0 & 2 & 4c+4\\8-8a & 6a-6 & 4+2ac\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$so\begin{bmatrix}1 & 0 & c+1\\0 & 1 & 2c+2\\4-4a & 3a-3 & 2+ac\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

on comparing terms, $a = 1, c = -1$. **Ans.**

4. The elements of the determinant of order 3 × 3 are {0, 1}. Then find the maximum and minimum values.

   Solution: $|A|_{maximum}$, when the diagonal elements are minimum of {0, 1} and other elements are maximum of {0, 1}

   So $|A|_{minimum} = -|A|_{maximum}$

   $$|A|_{maximum} = \begin{vmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{vmatrix} = 0 - 1(0-1) + 1(1-0)$$

   $|A|_{maximum} = 2 \; and \; |A|_{minimum} = -2$ **Ans.**

5. For any non-zero square matrix A of order n, show that $det[adj(adjA)] = |A|^{(n-1)^2}$

   Solution: We know that $A(adjA) = |A|I_n \; and \; |adjA| = |A|^{n-1}$

   Replace $A \; by \; (adjA)$

   $(adjA)[adj(adjA)] = |adjA|I_n$

   $(adjA)[adj(adjA)] = |A|^{n-1}I_n$

   $so\;(A.adjA)[adj(adjA)] = A|A|^{n-1}I_n$ pre-multiply by A

   $so\;|A|I_n.[adj(adjA)] = |A|^{n-1}.A\;\{since\;A.I_n = A\}$

   $so\;I_n.adj(adjA) = |A|^{n-2}.A$ Multiply by $\frac{1}{|A|}$ to both sides

   $adj(adjA) = |A|^{n-2}.A$

   $|adj(adjA)| = ||A|^{n-2}\,A|$

   $|adj(adjA)| = (|A|^{n-2})^n|A| \;\; since\;|kA| = k^n|A|$

$|adj(adjA)| = |A|^{n^2-2n+1} = |A|^{(n-1)^2}$ **HP**

Five marks Questions

1. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, find A^{-1} and show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

Solution: $|A| = 0(0-1) - 1(0-1) + 1(1-0) = 2 \neq 0, so\ A$ is non-singular matrix and A^{-1}exist.

$adjA = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

$so\ A^{-1} = \frac{1}{|A|}(adj\ A) = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$. . . . Eq (1)

$Now\ A^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

So $A^2 - 3I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$. . . . Eq (2)

So from Eq (1) and (2), we have, $A^{-1} = \frac{1}{2}(A^2 - 3I)$ **HP.**

2. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ Show that $A^3 - 6A^2 + 5A + 11I = O$ and hence find A^{-1}

Solution: $A^2 = A.A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}.\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1.1 + 1.1 + 1.2 & 1.1 + 1.2 + 1(-1) & 1.1 + 1.(-3) + 1.3 \\ 1.1 + 2.1 + (-3).2 & 1.1 + 2.2 + (-3)(-1) & 1.1 + 2(-3) + (-3).3 \\ 2.1 + (-1).1 + 3.2 & 2.1 + (-1).2 + 3(-1) & 2.1 + (-1)(-3) + 3.3 \end{bmatrix}$

$So\ A^2 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$

$Now\ A^3 = A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}.\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$

So $LHS = A^3 - 6A^2 + 5A + 11I$

$$LHS = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$LHS = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} + \begin{bmatrix} -24 & -12 & -6 \\ 18 & -48 & 84 \\ -42 & 18 & -84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$LHS = \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10 & -13+18-5 & 58-84+15+11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Hence verified.

Now $A^3 - 6A^2 + 5A + 11I = O$ *Post multiply by* A^{-1}

$Or\ A^3.A^{-1} - 6A^2.A^{-1} + 5A.A^{-1} + 11I.A^{-1} = O.A^{-1}$

Or $A^2(A.A^{-1}) - 6A(A.A^{-1}) + 5I + 11A^{-1} = O$

Or $A^2I - 6AI + 5I + 11A^{-1} = O$

Or $A^2 - 6A + 5I + 11A^{-1} = O$

Or $11A^{-1} = 6A - 5I - A^2$

$$11A^{-1} = 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$11A^{-1} = \begin{bmatrix} 6-5-4 & 6-0-2 & 6-0-1 \\ 6-0+3 & 12-5-8 & -18-0+14 \\ 12-0-7 & -6-0+3 & 18-5-14 \end{bmatrix} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$So\ A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$ **Ans.**

3. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$. Find BA and use this to solve system of equations $y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17$.

Solution: $BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$$BA = \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

Since $|BA| = |6I| = 6^3|I| = 216 \neq 0 \; so \; B \neq 0 \; and \; A \neq 0$ and both are non-singular matrices and $B^{-1} and \; A^{-1}$ exist.

Now system of equations $y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17$

Or $x - y = 3, 2x + 3y + 4z = 17 \; and \; y + 2z = 7$ can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \; as \; BX = C$$

$$so \; X = B^{-1}C$$

Since $BA = 6I$

$$so \; B.\frac{A}{6} = I \; so \; B^{-1} = \frac{A}{6} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

So $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

So $x = 2, y = -1, z = 4$ **Ans.**

Practice Questions:

4. A mixture is to be made of three foods A, B and C. The three foods A, B, C contains nutrients P, Q R as shown in table below

	Ounces per pound		
Food	P	Q	R
A	1	2	5

B	3	1	1
C	4	2	1

How to form a mixture which will have 8 ounces of P, 5 ounces of Q and 7 ounces of R? Find by matrix method.

5. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix}$, find A^{-1} and using it solve the system of equations:

   $3x + 2y - 2z = 3; x + 2y + 3z = 6; 2x - y + z = 2.$

6. A company produces three products every day. Their production on a certain day is 45 tons. It is found that third product exceeds the production of first by 8 tons while the total production of first and third product is twice the production of second product. Determine the production level of each product using matrix method.

CASE STUDY QUESTIONS

1. On her birthday, Seema decided to donate some money to children of orphanage home. If there were 8 children less, everyone will get Rs 10 more. However, if there are 16 children more, everyone will get Rs 10 less. Let the number of children be $'x'$ and the amount distributed by Seema to each child be $Rs\,'y'$.

Based on this information, answer any four of the following:

I. The total amount of money distributed by Seema is

a. $Rs\ (x + y)$, b. $Rs\ (y - x)$, c. $Rs\ (x \times y)$, d. $Rs\ (x - y)$

II. The system of equations formed, from the given condition is:

a. $5x - 4y = 40\ and\ 8y - 5x = 80$, b. $5x + 4y = 40\ and\ 8y + 5x = 80$,

c. $5x + 4y = 40\ and - 8y + 5x = 80$, d. $5x - 4y = 40\ and\ 5x - dy = 80$

III. The system of equations in matrix forms is:

a. $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$, b. $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 80 \\ 40 \end{bmatrix}$

c. $\begin{bmatrix} -5 & 8 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [40 \quad 80]$, d. $\begin{bmatrix} 5 & -8 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [80 \quad 40]$

IV. The value of $'x'$ and $'y'$ respectively are:

a. 32, 35 b. 30, 35 c. 35, 32 d. 32, 30

V. The adjoint of matrix $\begin{bmatrix} -5 & 8 \\ 5 & -4 \end{bmatrix}$ is:

a. $\begin{bmatrix} -4 & -8 \\ -5 & 5 \end{bmatrix}$, b. $\begin{bmatrix} 5 & -8 \\ -5 & 4 \end{bmatrix}$, c. $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}$, d. $\begin{bmatrix} 5 & 8 \\ 5 & 4 \end{bmatrix}$

Answers: I(c). II (a), III(a), IV (d), V(a)

2. A trust having fund of Rs 30000 to invest into two different types of bonds. The first bond pays 5% interest per annum which will be given to an orphanage and the second bond pays 7% interest per annum which will be given to 'Cancer Aid Society' an NGO. The trust wishes to divide Rs 30000 among two types of bonds in such a way that they earn an annual total interest of Rs 1800 each.

Based on above information, answer any four of the following:

I. Assuming that Rs x are invested in first bond, what amount is invested in second bond?

a. $Rs(x - 30000)$, b. $Rs(30000 - x)$,c. $Rs\ (30000 + x)$, d. $Rs\ (30000x)$

II. If the amount invested in second bond is Rs y, then the system of equations formed as:

a. $x + y = 30000\ and\ 5x + 7y = 1800$,

b. $x + y = 30000\ and\ 5x + 7y = 18000$

c. $x + y = 30000\ and\ 5x + 7y = 180000$

d. $x + y = 300000\ and\ 5x + 7y = 18000$

III. The system of equations in matrix form is:

a. $\begin{bmatrix} 1 & 1 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 1800 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 180000 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} [x \quad y] = [30000 \quad 1800]$ d. $\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix} [x \quad y] = \begin{bmatrix} 30000 \\ 18000 \end{bmatrix}$

IV. The values of 'x' and 'y' respectively are:

a. 12000, 18000. b. 15000, 15000.

c. 18000, 12000. d. 20000, 10000

V. The adjoint of matrix $\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}$ is:

a. $\begin{bmatrix} 7 & 5 \\ 1 & 1 \end{bmatrix}$, b. $\begin{bmatrix} -1 & 5 \\ 1 & -7 \end{bmatrix}$, c. $\begin{bmatrix} 7 & -1 \\ -5 & 1 \end{bmatrix}$, d. $\begin{bmatrix} 7 & -5 \\ -1 & 1 \end{bmatrix}$

Ans: I(b), II(c), III(b), IV(b), V(c)

3. The management committee of a residential society decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean.

Below are three statements given by committee.

A. The sum of all awardee is 12.

B. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33.

C. The sum of number of awardees for honesty and supervision is twice the number of awardees for helping others.

Based on above information, answer any four of the following questions:

I. The system of equations for the statements A, B and C respectively are:

a. $x + y + z = 12; \quad 3x + 2(y + z) = 33; \quad x + z = 2y$

b. $x + y + z = 12; \quad 3y + 2(x + z) = 33; \quad y + z = 2x$

c. $x + y + z = 12; \quad 3z + 2(x + y) = 33; \quad x + z = 2y$

d. $x + y + z = 12; \quad 2x + 3(y + z) = 33; \quad x + z = 2y$

II. The above group of equations in matrix form PX=Q, is written as:

a. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$, b. $\begin{bmatrix} 1 & 1 & 1 \\ -3 & 2 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 2 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$, d. $\begin{bmatrix} 1 & 1 & 1 \\ -3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 33 \\ 12 \end{bmatrix}$

III. $|P|$ is equal to:

a. 3, b. 2, c. 12, d. -14

IV. $adj(P)$ is:

a. $\begin{bmatrix} 4 & -3 & -1 \\ 4 & 0 & -4 \\ 4 & -3 & 5 \end{bmatrix}$, b. $\begin{bmatrix} 0 & 1 & -1 \\ -2 & 0 & 2 \\ 4 & -1 & -1 \end{bmatrix}$,

c. $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 3 \\ -3 & 1 & -1 \end{bmatrix}$, d. $\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$

V. The number of awardees of each category are:

a. $x = 3;\ y = 5;\ z = 4$, b. $x = 3; y = 4; z = 5$,

c. $x = 5; y = 4; z = 3$, d. $x = 4; y = 5; z = 3$

Ans: I(d), II (c) , III(a), IV(d), V(b)

BY A. K. GUPTA, PGT(MATHS) KV VV NAGAR

CH 5: CONTINUITY AND DIFFERENTIABILITY

1 MARK QUESTIONS

1. If $f(x) = |cosx - sinx|$, the find $f'\left(\frac{\pi}{3}\right)$

Ans. If $f(x) = |cosx - sinx|$

$if\ \frac{\pi}{4} < x < \frac{\pi}{2}$ then $sinx > cosx$

f(x) = - (cosx -sinx)

f′ (x) = sinx +cosx and $f'\left(\frac{\pi}{3}\right) = \frac{\sqrt{3+1}}{2}$

2. Find the set of points where f(x) is differentiable $f(x) = |sinx|$

Ans Not differentiable at x= nπ, $n\epsilon Z$

3. Differentiate $\tan(x^{\circ}+45^{\circ})$

Ans. Y=$\tan(x^{\circ}+45^{\circ})$

$=tan\left(\frac{\pi}{180}x + \frac{\pi}{4}\right)$

On diff.

$=\frac{\pi}{180}sec^{2(}\frac{\pi}{180}x + \frac{\pi}{4})$

$=\frac{\pi}{180}sec^{2}(x^{\circ} + 45^{\circ})$

4. If $y = \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} + \frac{1}{1+x^{b-a}+x^{c-a}}$, find $\frac{dy}{dx}$

Ans.

$$y = \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{b-c} + x^{a-c}} + \frac{1}{1 + x^{b-a} + x^{c-a}}$$

Y = $\frac{x^a+x^b+x^c}{x^a+x^b+x^c}$ =1,

$\frac{dy}{dx} = 0$

5. If f(0) =f(1)=0 and f ′(1)=2 and $y = f(e^x)e^{f(x)}$, find $\frac{dy}{dx}$ at x = 0

Ans. $y = f(e^x)e^{f(x)}$,

$$\frac{dy}{dx} = f'(e^x)e^x.e^{f(x)} + f(e^x)e^{f(x)}f'(x)$$

At x=0

$$\frac{dy}{dx} = f'(e^0)e^0.e^{f(0)} + f(e^0)e^{f(0)}f'(0)$$

$$\frac{dy}{dx} = f'(1).e^0 + f(1)e^0f'(0)$$

=2+0

Ans =2

2 MARKS QUESTIONS

6.

If $y^x = e^{y-x}$,then prove that $\frac{dy}{dx} = \frac{(1+logy)^2}{logy}$.

Ans. $y^x = e^{y-x}$

Taking log

x logy = (y-x)

$x = \frac{y}{1+logy}$

Diff. with respect to y

$\frac{dx}{dy} = \frac{logy}{(1+logy)^2}$

$\frac{dy}{dx} = \frac{(1+logy)^2}{logy}$.

7. Find the derivative of $cos^{-1}(2x^2 - 1)$ with respect to $cos^{-1}x$

Ans. u= $cos^{-1}(2x^2 - 1)$, $v = cos^{-1}x$

$let\ x = cos\theta$

u= $cos^{-1}(2cos^2\theta - 1)$, v = $cos^{-1}x$

=$cos^{-1}(cos2\theta)$

=2θ =2$cos^{-1}x$

$\frac{du}{dx} = \frac{-2}{\sqrt{1-x^2}}$ $\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}}$

$\frac{du}{dv} = 2$

8. If y = f($\frac{2x-1)}{x^2+1)}$

and $f'(x) = sinx^2$,find $\frac{dy}{dx}$

Ans y = f($\frac{2x-1)}{x^2+1)}$

$z = \frac{2x-1)}{(x^2+1)}$

y = f(z)

$\frac{dy}{dx} = f'^{(}(z)\frac{d}{dx}(\frac{2x-1}{x^2+1})$

$= sinz^2 \left(\frac{1+x-x^2}{(x^2+1)^2}\right)$

$= 2sin\left(\frac{2x-1)}{(x^2+1)}\right)^2 \left(\frac{1+x-x^2}{(x^2+1)^2}\right)$

9. If (x -y)$e^{\frac{x}{x-y}} = a$,

Prove that $y\frac{dy}{dx}$ + x = 2y

Ans If (x -y)$e^{\frac{x}{x-y}} = a$,
Taking log
$\log(x-y) + \frac{x}{x-y} = loga$
on diff
$\frac{1}{x-y}(1-\frac{dy}{dx})$ + $\frac{(x-y)-x(1-\frac{dy}{dx})}{(x-y)^2} = 0$
On simplifying we will get
$y\frac{dy}{dx}$ + x = 2y

10. If $y = cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ then find $\frac{dy}{dx}$
$y = cos^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$
$y = cos^{-1}\left(\frac{2.2^x}{1+\ 2^{2x}}\right)$
Y = $2tan^{-1}2^x$

$$\frac{dy}{dx} = \frac{2.2^x\ log2}{1+\ 4^x}$$

3 MARKS QUESTIONS

11. If $\sqrt{1-\ x^2} + \sqrt{1-\ y^2} = a(x-y)$ then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Ans. Put x = sinA , x = sinB ,we get
$\sqrt{1-sin^2A} + \sqrt{1-sin^2B} = a(sinA - \sin B)$

$$\frac{cosA + cosB}{sinA - sinB} = a$$

Cot(A -B/2) =a
A – B = 2 $cot^{-1}a$
$sin^{-1}x + \ sin^{-1}y = 2cot^{-1}a$
On Diff

$$\frac{dy}{dx} = \sqrt{\frac{1-\ y^2}{1-\ x^2}}$$

12. If $y = \ x^x$ then prove that

$$\frac{d^2y}{d^2x} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

Ans. If $y = \ x^x$, prove that
logy =xlogx
on diff.

$\frac{dy}{dx} = y(1 + logx)$ ---------------------1

Again diff.

$$\frac{d^2y}{d^2x} = y\frac{1}{x} + \frac{dy}{dx}(1 + logx)$$

Using -----1

$$\frac{d^2y}{d^2x} = \frac{y}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2$$

$$\frac{d^2y}{d^2x} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

13. If $x^m\, y^n = (x + y)^{m+n}$ prove that

$\frac{dy}{dx} = \frac{y}{x}$ ii. $\frac{d^2y}{d^2x} = 0$

Ans. $x^m\, y^n = (x + y)^{m+n}$

mlogx + nlogy = (m +n) log(y +x)

diff.

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m + n}{y + x}(\frac{dy}{dx} + 1)$$

On simplifying

$$\frac{(nx - my)}{y(x + y)}\frac{dy}{dx} = \frac{(nx - my)}{x(y + x)}$$

$\frac{dy}{dx} = \frac{y}{x}$

Again differentiating

$\frac{d^2y}{d^2x} = 0$

14. If $\sqrt{1 - x^6} + \sqrt{1 - y^6}$ = a$(x^3 - y^3)$

,prove that $\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1 - y^6}{1 - x^6}}$

Ans. Put x^3 = sinA , y^3 = sinB ,we get

$$\sqrt{1 - sin^2A} + \sqrt{1 - sin^2B} = a(sinA - \sin B)$$

$$\frac{cosA + cosB}{sinA - sinB} = a$$

Cot(A -B/2) =a

A – B = 2 $cot^{-1}a$

$sin^{-1}x^3 + sin^{-1}y^3 = 2cot^{-1}a$

On Diff

$$\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1 - y^6}{1 - x^6}}$$

15. If $y = cos^{-1}\left(\frac{2x - 3\sqrt{1 - x^2}}{\sqrt{13}}\right)$,find $\frac{dy}{dx}$

Ans. $y = cos^{-1}\left(\frac{2x - 3\sqrt{1 - x^2}}{\sqrt{13}}\right)$

Let x = cosθ , cos$\alpha = \frac{2}{\sqrt{13}}$

Y = cos^{-1}(cosθ cos$\alpha - sin\alpha$ sinθ)

Y= θ + α

Y =$cos^{-1}x + \alpha$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$

16. Prove that sgn(x -1) is not differentiable at x = 1.

Ans Prove that sgn(x -1) is not differentiable at x = 1.

R.H.D=$\log_{h\to 0}\frac{f(1+h)-f(1)}{h}$

=$\log_{h\to 0}\frac{sgn(1+h-1)-sgn(0)}{h}$

=$\log_{h\to 0}\frac{1-0}{h}$

=∞

L.H.D=$\log_{h\to 0}\frac{f(1-h)-f(1)}{-h}$

$= \log_{h\to 0}\frac{sgn(1-h-1)-sgn(0)}{h}$

$= \log_{h\to 0}\frac{-1}{h}$

= −∞

Not diff .at x=1

5 MARKS QUESTIONS

17. If x = sint and y = sinpt then prove that

$(1-x^2)\frac{d^2y}{d^2x} - x\frac{dy}{dx} + p^2y$ = 0

Ans. x = sint and y = sinpt

$\frac{dx}{dt} = cost, \quad \frac{dy}{dt} = pcospt$

$\frac{dy}{dx} = \frac{pcospt}{cost}$

Again diff.

$\frac{d^2y}{d^2x} = \frac{-p^2sinptcost + pcospt.sint}{cos^3t}$

On substituting values in

$L.H.L = (1-x^2)\frac{d^2y}{d^2x} - x\frac{dy}{dx} + p^2y$ =0

18.

If $x = sin\left(\frac{1}{a}\ logy\right)$

show that $(1+\ x^2)\frac{d^2y}{d^2x} - x\frac{dy}{dx} -\ a^2y = 0$

Ans. $x = sin\left(\frac{1}{a}\ logy\right)$

$logy = asin^{-1}x$
Diff.
$y_{1=} \dfrac{a}{\sqrt{1-x^2}}$
$\sqrt{1-x^2}y_{1=} a$y
$(1-x^2)y_{1=}^2 a^2y^2$
Again Diff.
$(1-x^2).2y_1\,y_2 \;-2xy_1^2 = 2a^2yy_1$

$$(1+x^2)\frac{d^2y}{d^2x} - x\frac{dy}{dx} - a^2y = 0$$

19. If $f(x) = \begin{cases} \frac{\sin(a+1)x+2sinx}{x} & if\ x<0 \\ 2 & if\ x=0 \\ \frac{\sqrt{1+bx}-1}{x} & if\ x>0 \end{cases}$

Ans. $f(x) = \begin{cases} \dfrac{\sin(a+1)\,x+2sinx}{x} & if\ x<0 \\ 2 & if\ x=0 \\ \dfrac{\sqrt{1+bx}-1}{x} & if\ x>0 \end{cases}$

f(0) =2
R.H.L=$\log_{h\to0}\frac{\sqrt{1+bh}-1}{h}$
$= \log_{h\to0}\dfrac{(\sqrt{1+bh}-1)(\sqrt{1+bh}+1)}{h(\sqrt{1+bh}+1)}$

$= \log_{h\to0}\dfrac{(1+bh-1)}{h(\sqrt{1+bh}+1)}$
=$\frac{b}{2}$
L.H.L=
$= \log_{h\to0}\dfrac{\sin(a+1)\,(-h)+2\sin(-h)}{-h}$

$= \log_{h\to0}\dfrac{\sin(a+1)\,(h)}{h} + \dfrac{2sinh}{h}$
$= \log_{h\to0}\dfrac{(\mathrm{a}+1)\sin(a+1)\,(h)}{(a+1)h} + \log_{h\to0}\dfrac{2sinh}{h}$

=a +3

$\dfrac{b}{2} = a+3 = 2$

a=-1 , b=4

20. Find the values of p and q for which

$$f(x) = \begin{cases} \frac{1-\ sin^3 x}{3cos^2 x} & if\ x\ < \frac{\pi}{2} \\ p & if\ x = \frac{\pi}{2} \\ \frac{q(1-sinx)}{(\pi-2x)^2} & if\ x\ < \frac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}$

Ans.

$$f(x) = \begin{cases} \frac{1-\ sin^3 x}{3cos^2 x} & if\ x\ < \frac{\pi}{2} \\ p & if\ x = \frac{\pi}{2} \\ \frac{q(1-sinx)}{(\pi-2x)^2} & if\ x\ < \frac{\pi}{2} \end{cases}$$

f($\frac{\pi}{2}$) =p

L.H.L = $\log_{h\to 0} \frac{1-\ sin^3(\frac{\pi}{2}-h)}{3cos^2(\frac{\pi}{2}-h)}$

$= \log_{h\to 0} \frac{1-\ cos^3 h}{3sin^2(h)}$

$=\log_{h\to 0} \frac{(1-\cosh)(\ 1+cos^2 h+cosh)}{3(1-COS^2(h)}$

$=\log_{h\to 0} \frac{(\ 1+cos^2 h+cosh)}{3(1+\cosh)} = \frac{1}{2}$

R.H.L $=\log_{h\to 0} \frac{q(1-sin(\frac{\pi}{2}+h))}{(\pi-2(\frac{\pi}{2}+h))^2}$

$= \log_{h\to 0} \frac{q(1-cosh)}{4h^2}$

$= \log_{h\to 0} q\left(\frac{sin\frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{8} = \frac{q}{8}$

$\frac{q}{8} = p = \frac{1}{2}$

21. Find the values of p and q so that

$$f(x) = \begin{cases} x^2+3x+p & if\ x\ \le 1 \\ qx+2 & if\ x\ > 1 \end{cases}$$

Ans. $f(x) = \begin{cases} x^2+3x+p & if\ x\ \le 1 \\ qx+2 & if\ x\ > 1 \end{cases}$

R.H.D=$\log_{h\to 0}\frac{f(1+h)-f(1)}{h}$
$=\log_{h\to 0}\frac{q(1+h)-[1+3+p]}{h}$
$=\log_{h\to 0}\frac{qh+q-2-p]}{h}$
Limit will exist only when
q-2-p=0
p-q=2
R.H.D=$\log_{h\to 0}\frac{qh+0]}{h}$
=q

l.H.D=$\log_{h\to 0}\frac{f(1-h)-f(1)}{-h}$
l.H.D=$\log_{h\to 0}\frac{[(1-h)^2+3(1-h)]+p-[1+3+p]}{-h}$
l.H.D=$\log_{h\to 0}\frac{h(h-5)}{-h}$ =5

p=3
q=5

22. If (ax + b)$e^{\frac{y}{x}} = x$ prove that

$$x^3\frac{d^2y}{d^2x} = \left(x\frac{dy}{dx} - y\right)^2$$

Ans. (ax + b)$e^{\frac{y}{x}} = x$

$e^{\frac{y}{x}} = \frac{x}{(\text{ax + b})}$

Taking log

$\frac{y}{x} = logx - \log(\text{ax + b})$

On diff.

$\frac{x\frac{dy}{dx}-y}{x^2} = \frac{1}{x} - \frac{b}{\text{ax + b}}$

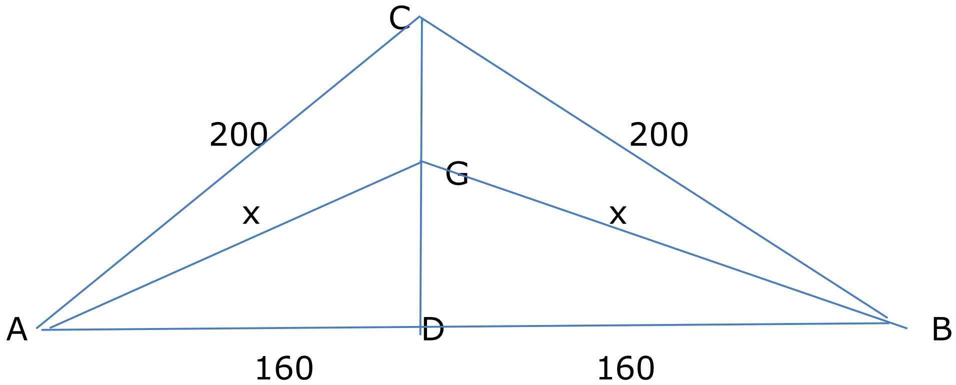
$x\frac{dy}{dx} - y = \frac{ax}{\text{ax + b}}$ -----1

Again diff. and using eq.1
We will get

$$x^3\frac{d^2y}{d^2x} = \left(x\frac{dy}{dx} - y\right)^2$$

CASE STUDY BASED QUESTIONS

A firm has a branch store in each of the three cities A, B, C. A and B are 320km apart and C is 200km from each of them .A godown G is to be built equidistant from A and B . If the distances from the godown to each of the cities is minimum .



Based on the above information answer the following questions:

(i) What is the value of CD ?

(a) 200 (b) 160 (c) 120 (d) 140

(ii) What is the value of GD ?

(a) $\sqrt{x^2-(160)^2}$ (b) $\sqrt{x^2+(160)^2}$ (c) $\sqrt{x^2-160}$ (d) $\sqrt{x^2+160}$

(iii) If y =GA + GB +GC, then what is the value of x when $\frac{dy}{dx}=0$?

(a) $\frac{320}{\sqrt{2}}$ (b) $\frac{320}{\sqrt{3}}$ (c) $\frac{320}{\sqrt{5}}$ (d) $\frac{320}{\sqrt{6}}$

(iv) What is the value of $\frac{d^2y}{d^2x}$?

(a) $\frac{x}{2\sqrt{x^2-(160)^2}}$ (b $\frac{2x^2}{2\sqrt{x^2-(160)^2}}$ (c) $\frac{x}{\sqrt{x^2-(160)^2}}$ (d) None of these

(v) What is the value of $\frac{dy}{dx}$?

(a) $\frac{1}{2\sqrt{x^2-(160)^2}}$ (b) $2-\frac{2x}{2\sqrt{x^2-(160)^2}}$ (c) $2+\frac{2x}{2\sqrt{x^2-(160)^2}}$ (d) $\frac{2x}{2\sqrt{x^2-(160)^2}}$

Ans.
- i. c
- ii. a
- iii. b
- iv. b
- v. d

BY MR L.S. RAWAT, PGT(MATHS) KV AFS, BARODA

CH 6: APPLICATIONS OF DERIVATIVES

1- MARK QUESTIONS

1. The maximum value of sinx + cosx is _____.
2. Find the point of inflection for the function $f(x) = x^3$.
3. Minimum value of f if $f(x) = sinx$ in the interval [-π/2, π/2] is _____.
4. If f is a function such that $f'(c) = 0, f''(c) = 0$ and $f'''(c) > 0$ then c is a point of ___________.
5. Show that the function f(x) = e^{2x} is strictly increasing on R.
6. On which point the line y = x + 1 is a tangent to the curve $y^2 = 4x$
7. Find the slope of the normal to the curve $y = 1/x$ at the point (3,1/3)
8. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point where the curve cuts y-axis.
9. Write the interval for which the function f(x) = 1/x is strictly decreasing
10. It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the value of a.

HINTS/ANSWERS TO 1 MARKS QUESTIONS

1) $\sqrt{2}$ 2) x=0 3) -1 4) Local Minima 5) As $e^{2x} > 0$ for xϵR.
6) for the curve $\frac{dy}{dx} = \frac{2}{y}$, slope of given line = slope of tangent $\Rightarrow 1 = \frac{2}{y} \Rightarrow$ y=2 which gives $y^2 = 4x \Rightarrow 2^2 = 4x \Rightarrow x = 1$, hence the required point is (1,2)
7) $\frac{dy}{dx} = -\frac{1}{x^2}$, Slope of Normal at (3,1/3) = $-1/\left(\frac{dy}{dx}\right) = -1/\left(-\frac{1}{x^2}\right) = +x^2$ $= (3)^2 = 9$
8) Curve cuts y-axis where x=0 which gives y=0-0+1=1, so we need to find the slope at the point (0,1), now $\frac{dy}{dx} = 3x^2 - 1$, so the required slope at (0,1) = $3(0)^2 - 1 = -1$
9) Here f′(x) = $-1/x^2$,which is $-ve$ for all real values of x in its domain,so f(x) is strictly decreasing.
10) Here f′(x) =$4x^3$-124x+a, Since the function f attains its maximum value on the interval [0, 2] at x = 1, therefore $f'(1) = 0 \Rightarrow 4 - 124 + a = 0$ $\Rightarrow$ a=120

2-MARKS QUESTIONS

1. Show that the local maximum value of $x + \frac{1}{x}$ is less than local minimum value.

Solution: Let $y = x + \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$

For Critical points,

$\frac{dy}{dx} = 0 \; gives \; x^2 = 1 \Rightarrow \mathrm{x} = \pm 1$

Now,

$\frac{d^2y}{d^2x} = +\frac{2}{x^3}$, therefore $\frac{d^2y}{d^2x}$ (at x = 1) > 0 and $\frac{d^2y}{d^2x}$ (at x = −1) < 0

Hence local maximum value of y is at x = −1 and the local maximum value = – 2.

Local minimum value of y is at x = 1 and local minimum value = 2.

Clearly, local maximum value (−2) is less than local minimum value 2.

2. Find a point on the curve y = $(x - 2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

Solution:

If a tangent is parallel to the chord joining the points (2, 0) and (4, 4), then

The slope of the tangent = The slope of the chord

Here, The slope of the chord $= \frac{4-0}{4-2} = 2$

Now, the slope of the tangent to the given curve at a point (x, y) is given by $\frac{dy}{dx} = 2(x - 2)$

Since the slope of the tangent = slope of the chord, we have:
2(x-2) =2, which gives x=3

When x=3, y=$(3-2)^2$=1

Hence, the required point is (3, 1).

3. Show that the function f(x) = $4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

Solution: Here f (x) = $4x^3 - 18x^2 + 27x - 7$
f ′ (x) = $12x^2 - 36x + 27 = 3(4x^2 - 12x + 9) = 3(2x - 3)^2$
f ′ (x) = 0 ⇒ x =3/2 (critical point)
Since f ′ (x) > 0 for all x <3/2 and x>3/2
Hence x =3/2 is a point of inflexion i.e., neither a point of maxima nor a point of minima.
x =3/2 is the only critical point, and f has neither maxima nor minima

4. Find all the points of local maxima and local minima of the function $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$**.**

Solution: Here

$f'(x) = -3x^3 - 24x^2 - 45x$

$= -3x(x^2 + 8x + 15)$

$= -3x(x + 5)(x + 3)$

For Critical points,

$f'(x) = 0 \Rightarrow x = -5, x = -3, x = 0$

Now,

$f''(x) = -9x^2 - 48x - 45$

$= -3(3x^2 + 16x + 15)$

$f''(0) = -45 < 0$. Therefore, x = 0 is point of local maxima

$f''(-3) = 18 > 0$. Therefore, x = −3 is point of local minima

$f''(-5) = -30 < 0$. Therefore x = −5 is point of local maxima.

5. Find the least value of 'a' such that $f(x) = x^2 + ax + 1$ **is increasing on [1,2].**

Solution:

Here $f'(x) = 2x + a$

Here $x \in [1,2] \Rightarrow 1 \leq x \leq 2 \Rightarrow 2 \leq 2x \leq 4 \Rightarrow 2 + a \leq 2x + a \leq 4 + a$

$\Rightarrow 2 + a \leq f'(x) \leq 4 + a$

For $f(x)$ to be strictly increasing $f'(x) \geq 0$ for that $2 + a \geq 0 \Rightarrow a \geq -2$

Hence the least value of a such that f(x) is increasing in the given interval is -2.

3 MARKS QUESTIONS

1. Find the intervals in which the function f(x)=$-2x^3-9x^2-12x+1$ is strictly increasing or decreasing.

Solution:

We have,

$f(x)=-2x^3-9x^2-12x+1$

which gives

$f'(x)=-6x^2-18x-12)=-6(x^2-3x-2)=-6(x+1)(x+2)$

For critical points

$f'(x)=0 \Rightarrow -6(x+1)(x+2)=0 \Rightarrow x=-1$ or $x=-2$

Points $x=-1$ and $x=-2$ divide the real line into three disjoint intervals i.e $(-\infty,-2),(-2,-1)$ and $(-1,\infty)$ In intervals $(-\infty,-2)$ and $(-1,\infty)$ i.e., when $x<-2$ and $x>-1$, $f'(x)=-6(x+1)(x+2)<0$

$\therefore f$ is strictly decreasing for $x<-2$ and $x>-1$.

Now, in interval $(-2,-1)$ i.e., when $-2<x<-1$,

$f'(x)=-6(x+1)(x+2)>0$

$\therefore f$ is strictly increasing for $-2<x<-1$

2. Find the maximum and minimum values of f(x) =x+sin2x in the interval [0, 2π].

Solution: We have f(x) =x+sin2x.

So, $f'(x)=1+2\cos 2x$. For stationary points, we have $f'(x)=0$

$\Rightarrow 1+2\cos 2x=0 \Rightarrow \cos 2x=-\frac{1}{2} \Rightarrow 2x=\frac{2\pi}{3}$ or $2x=\frac{4\pi}{3} \Rightarrow x=\frac{\pi}{3}$ or $x=\frac{2\pi}{3}$

Now we have,

$f(0)=0+sin0=0$

$f(\frac{\pi}{3})=\frac{2\pi}{3}+sin\frac{\pi}{3}=\frac{2\pi}{3}+\frac{\sqrt{3}}{2}$

$f(\frac{2\pi}{3})=\frac{2\pi}{3}+sin\frac{4\pi}{3}=\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$

$f(2\pi)=2\pi+sin4\pi=2\pi+0=2\pi.$

Of these values, the maximum value is 2π and the minimum value is 0.

Thus, the maximum value of f(x) is $2\pi$and the minimum value is 0.

3. Find the maximum and minimum values of f(x) =$x^{\frac{1}{x}}$ and hence deduce that $e^{\pi} > \pi^{e}$.

Solution: y=$x^{\frac{1}{x}}$

log y=$\frac{1}{x}\log x$

Differentiate both side w.r.t to x

$\frac{1}{y}\frac{dy}{dx} = \frac{x.\frac{1}{x} - logx}{x^2} \Rightarrow \frac{dy}{dx} = \frac{y[1-logx]}{x^2}$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}\frac{[1-logx]}{x^2} + y\frac{x^2\left(0-\frac{1}{x}\right) - (1-logx)2x}{x^4}$$

$$= \frac{dy}{dx}\frac{[1-logx]}{x^2} + y\frac{[-1-2(1-logx)]}{x^3}$$

For max or min $\frac{dy}{dx} = 0 \Rightarrow \frac{y[1-logx]}{x^2} = 0$

y (1-log x) =0

1-log x=0

logx=1

x=e

$\frac{d^2y}{dx^2} = 0 + \frac{e^{\frac{1}{e}}[-1-0]}{e^3}$ $\left[as\ at\ x = e, \frac{dy}{dx} = 0\ and\ log_e = 1\right]$

$= -ve$

$\therefore y\ is\ \max when\ x = e\ and\ \max value\ of\ y = e^{\frac{1}{e}}$

Now, it is known that $x^{\frac{1}{x}}$ is greater than any value of x except x=e.

Hence, the value of $x^{\frac{1}{x}}$ is greater at x=e than at x=π

i.e., $e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$.

$\Rightarrow \left(e^{\frac{1}{e}}\right)^{\pi e} > \left(\pi^{\frac{1}{\pi}}\right)^{\pi e}$

$\Rightarrow e^{\pi} > \pi^{e}$.

4. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 16 m, find the dimensions of the rectangle that will produce the largest area of the window.

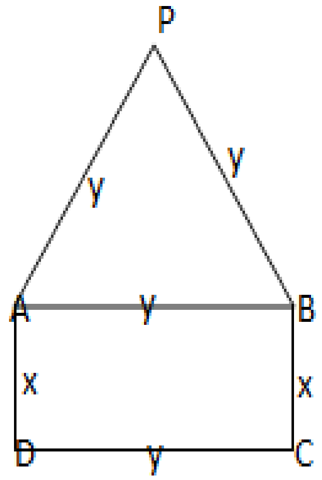
Solution:

Let the length and width of the window are x m and y m respectively.

Perimeter of window=16 m.

Therefore, $2x + 3y = 16 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

Let area of the window is A.



$A = xy + \frac{\sqrt{3}}{4}y^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$

$A = 8y - \frac{3}{2}y^2 + \frac{\sqrt{3}}{4}y^2 \ldots\ldots\ldots\ldots\ldots (3)$

Differentiating (3) w.r.t. y

$$\frac{dA}{dy} = 8 - 3y + \frac{\sqrt{3}}{2}y$$

For A to be maximum, $\frac{dA}{dy} = 0 \Rightarrow y = \frac{16}{6-\sqrt{3}}$

$Now, \frac{d^2A}{dy^2} < 0$ at y $= \frac{16}{6-\sqrt{3}}$

$Therefore,$ A is maximum,when y $= \frac{16}{6-\sqrt{3}} m$.

5. A manufacture can sell x items at a price of Rs $\left(5 - \frac{x}{100}\right)$ each the cost price of x items Rs ($\frac{x}{5} + 500$). Find the number of items he should sell to earn maximum profit.

S.P. of x items = x (5 –x/100) =5x – x^2/100

C.P. of x items = x/5 +500

Profit =P(x) = S.P -C. P. = (5x–x^2/100) – (x/5 +500)

P(x) =24x/5 –x^2/100–500 ⇒ For maximum profit

$$\frac{dP}{dx} = 0$$

⇒24/5–2x/100 =0

⇒x = 240

Now, at x=240, $\frac{d^2p}{dx^2} = -\frac{1}{50} < 0$,

Profit will be maximum if the number of items to be sold are 240.

5-MARKS QUESTIONS

1. Find the maximum value of $2x^3 - 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in [−3, −1].

Solution:

Let $f(x) = 2x^3 - 24x + 107$.

So, $f'(x) = 6x^2 - 24 = 6(x^2 - 4)$

Now, for critical points, $f'(x)=0$ i.e. $6(x^2 - 4)=0$

which gives x=2 or x=-2

We first consider the interval [1, 3].

Then, we evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of the interval [1, 3].

$f(2) = 2(8) - 24(2) + 107 = 16 - 48 + 107 = 75$

$f(1) = 2(1) - 24(1) + 107 = 2 - 24 + 107 = 85$

$f(3) = 2(27) - 24(3) + 107 = 54 - 72 + 107 = 89$

Hence, the absolute maximum value of f(x) in the interval [1, 3] is 89 occurring at x = 3.

Next, we consider the interval [−3, −1]

Evaluate the value of f at the critical point $x = -2 \in [-3, -1]$ and at the end points of the interval [1, 3].

$f(-3) = 2(-27) - 24(-3) + 107 = -54 + 72 + 107 = 125$

$f(-1) = 2(-1) - 24(-1) + 107 = -2 + 24 + 107 = 129$

$f(-2) = 2(-8) - 24(-2) + 107 = -16 + 48 + 107 = 139$

Hence, the absolute maximum value of f(x) in the interval [−3, −1] is 139 occurring at x = −2.

2. Find the intervals in which following functions are strictly increasing or strictly decreasing $f(x) = (x+1)^3(x-3)^3$

Solution:

We have, f(x) = (x + 1)3 (x − 3)3

$f'(x) = 3(x + 1)^2 (x - 3)^3 + (x + 1)^3 .3(x - 3)^2$

$= 3(x + 1)^2 (x - 3)^2 [(x - 3) + (x + 1)]$

$= 3(x + 1)^2 (x - 3)^2 [2x - 2]$

$= 6(x + 1)^2 (x - 3)^2 [x - 1]$

Now,

f′(x) =0 gives x= −1,1,3

The points x =−1, x=1 and x=3 divide the real line into four disjoint intervals i.e., $(-\infty, -1)$, (−1, 1), (1, 3) and $(3, \infty)$.

In intervals $(-\infty, -1)$ and (−1, 1), $f'(x) = 6(x + 1)^2 (x - 3)^2 [x - 1] < 0$

∴ f is strictly decreasing in intervals and $(-\infty, -1)$ and (−1, 1).

In intervals (1, 3) and$(3, \infty)$, $f'(x) = 6(x + 1)^2 (x - 3)^2 [x - 1] > 0$

∴ f is strictly increasing in intervals (1, 3) and $(3, \infty)$.

Hence, f is Strictly increasing in (1,3), (3, ∞) and Strictly decreasing in $(-\infty, -1)$ and (−1, 1).

3. Prove that the curves x = y^2 and xy = k cut at right angles if $8k^2$ = 1.

Solution:

The equations of the given curves are $x = y^2$ and $xy = k$

Putting $x = y^2$ in $xy = k$, we get: $y^3 = k \Rightarrow y = k^{1/3}$, $which\ gives\ x = k^{2/3}$

Thus, the point of intersection of the given curves is $(k^{2/3}, k^{1/3})$.

Differentiating $x = y^2$ with respect to x, we have: $\frac{dy}{dx} = \frac{1}{2y}$

Therefore, the slope of the tangent to the curve $x = y^2$ at $(k^{2/3}, k^{1/3})$ is

$m_1 = \frac{dy}{dx} = \frac{1}{2y} = \frac{1}{2k^{1/3}}$

On differentiating $xy = k$ with respect to x, we have $x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

∴ Slope of the tangent to the curve xy=k at at $(k^{2/3}, k^{1/3})$ is

$m_2 = \frac{dy}{dx} = -\frac{y}{x} = -\frac{k^{\frac{1}{3}}}{k^{\frac{2}{3}}} = -\frac{1}{k^{\frac{1}{3}}}$

We know that two curves intersect at right angles if the tangents to the curves at the point of intersection i.e., at are perpendicular to each other. This implies that we should have the product of the tangents as − 1.

Thus, the given two curves cut at right angles if the product of the slopes of their respective tangents at is −1. Hence, the given two curves cut at right angles if

$m_1\ m_2 = -1$

$\Rightarrow \frac{1}{2k^{1/3}} \cdot \left(-\frac{1}{k^{1/3}}\right) = -1$

$\Rightarrow \frac{1}{2k^{2/3}} = 1$

$\Rightarrow 2k^{2/3} = 1$

$\left(2k^{2/3}\right)^3 = (1)^3$

$\Rightarrow 8k^2 = 1$

4. Find the equation of tangent to the curve y = $\sqrt{3x-2}$, which is parallel to the line 4x -2y + 5 =0.

Solution:

The equation of the given curve is y = $\sqrt{3x-2}$

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}}$$

The equation of the given line is 4x − 2y + 5 = 0.

⇒ y = 2x+ 5/2 (which is of the form y= mx + c)

∴ Slope of the line = 2

Now, the tangent to the given curve is parallel to the line 4x − 2y − 5 = 0 if the slope of the tangent is equal to the slope of the line. So,

$$\frac{3}{2\sqrt{3x-2}} = 2 \Rightarrow \sqrt{3x-2} = \frac{3}{4} \Rightarrow 3x - 2 = \frac{9}{16} \Rightarrow x = \frac{41}{48}$$

When $x = \frac{41}{48}$, y = $\sqrt{3(\frac{41}{48}) - 2} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Therefore, Equation of the tangent passing through the point (41/48,3/4) is given by,

$$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$$

$$\Rightarrow \frac{4y-3}{4} = 2\left(x - \frac{41}{48}\right)$$

$$\Rightarrow \frac{4y-3}{4} = 2\left(\frac{48x-41}{48}\right)$$

$$\Rightarrow 4y - 3 = \frac{48x-41}{6}$$

$$\Rightarrow 24y - 18 = 48x - 41$$

$$\Rightarrow 48x - 24y = 23$$

Hence, the equation of the required tangent is 48x - 24y = 23.

5. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

Solution:

let x and 28-x be the length the two pieces by which square and circle are

Area of square $=\left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$.

Area of circle formed $= \pi\left(\frac{28-x}{2\pi}\right)^2$ $[as\ 2\pi r = 28 - x]$

$$= \frac{(28-x)^2}{4\pi}$$

Combined Area$=\frac{x^2}{16} + \frac{(28-x)^2}{4\pi}$

So, $\frac{dA}{dx} = \frac{x}{8} - \frac{2(28-x)}{4\pi}$

Also, $\frac{d^2A}{dx^2} = \frac{1}{8} - \frac{1}{2\pi}$

For the point of maxima or minima

$$\frac{dA}{dx} = 0 \Rightarrow \frac{x}{8} - \frac{(28-x)}{2\pi} = 0$$

$$\Rightarrow x = \frac{112}{4+\pi}$$

So, the combined area is minimum at x$=\frac{112}{4+\pi}$ as $\frac{d^2A}{dx^2} > 0$

Hence the lengths of the wires should be $\frac{112}{4+\pi}\ and\ \frac{28\pi}{4+\pi}$

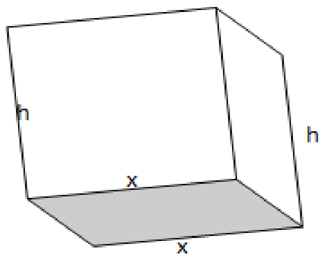
6. A open box, with a square base, is to be made out of a given quantity of metal sheet of area c^2.Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$.

Solution:

Let one edge of base of box is 'x' unit

And the height of the box is h, then

Area of the box= c^2 square Units.



$$x^2 + 4xh = c^2$$

Therefore $h = \frac{c^2 - x^2}{4x}$ .... (1)

Volume of box, $V = x^2 h$

Or $V = \frac{1}{4}(c^2 x - x^3)$ .... (2)

For V to be maximum, $\frac{dv}{dx} = 0 \Rightarrow x = \frac{c}{\sqrt{3}}$

Now, $\frac{d^2V}{dx^2} < 0$ at $x = \frac{c}{\sqrt{3}}$

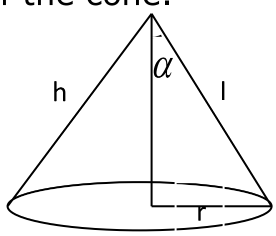
$\therefore V$ is maximum at $x = \frac{c}{\sqrt{3}}$

$Maximum\ volume = \frac{1}{4}(c^2 x - x^3) = \frac{1}{4}\left(\frac{c}{\sqrt{3}}\right)\left(c^2 - \frac{c^2}{3}\right) = \frac{\mathbf{c^3}}{\mathbf{6\sqrt{3}}}$

7. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\boldsymbol{sin^{-1}\frac{1}{3}}$

Solution:

Let r, h, l ,S and V be the radius, hight, slant hight, surface area and the volume of the cone.



S= $\pi rl + \pi r^2 \Rightarrow l = \frac{S - \pi r^2}{\pi r}$

$and\ V = \frac{1}{3}\pi r^2 h \Rightarrow V^2 = \frac{1}{9}\pi^2 r^4 h^2$ =z (say)

For Maximum volume, $\frac{dz}{dr} = 0$

$$\Rightarrow \frac{1}{9}(2rS^2 - 8S\pi r^3) = 0$$

$$\Rightarrow \frac{r}{l} = \frac{1}{3}$$

$$now \frac{d^2 z}{dr^2}(at S = 4\pi r^2) < 0$$

$$\therefore V^2 is\ maximum.$$

$\therefore V is\ maximum.$

Now $sin\,\alpha = \frac{r}{l} = \frac{1}{3} \Rightarrow \alpha = sin^{-1}(\frac{1}{3})$.

Hence the volume of right circular cone of given surface area is maximum when the semi vertical angle is $sin^{-1}\frac{1}{3}$.

8. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 \tan^2\alpha$

Solution: Let the radius of inscribed cylinder be x and its height be y.

$$Volume\ (V) = \pi x^2 y = \pi(h-y)^2 tan^2\alpha y$$

$$\frac{dV}{dy} = \pi\ tan^2\alpha(h^2 - 4hy + 3y^2$$

For max. or min. of V

$$\frac{dV}{dy} = 0 \Rightarrow 3y^2 - 4hy + h^2 = 0$$

$$3(y-h)(3y-h) = 0 \Rightarrow y = h, y = \frac{h}{3}$$

Since y=h is not possible, therefore at $y = \frac{h}{3}$ we have

$$\frac{d^2V}{dy^2} = 6y - 4h = 6\left(\frac{h}{3}\right) - 4 = -2h < 0$$

$$So, V\ is\ maximum\ at\ y = \frac{h}{3}$$

Now Maximum Volume at y=h/3 is given by

$$V = \pi\left(h - \frac{h}{3}\right)^2 tan^2\alpha \frac{h}{3}$$

$$= \frac{4}{27}\pi h^3 tan^2\alpha$$

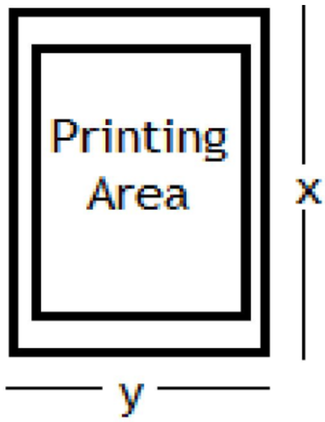
Hence Proved that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 \tan^2\alpha$

Case studies on Application of integrals

Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

Case study 6.1

Following is the pictorial description for a page



The total area of the page is 150 cm^2. The combined width of the margin at the top and bottom is 3 cm and the side 2 cm.

Using the information given above, answer the following:

i) The relation between x and y is given by

(a) (x-3) y=150

(b) xy = 150

(c) x(y-2) =150

(d) (x-2)(y-3) = 150

ii) The area of page where printing can be done, is given by

(a) xy

(b) (x+3)(y+2)

(c) (x-3)(y-2)

(d) (x-3)(y+2)

iii) The area of the printable region of the page, in terms of x, is

(a) $156 + 2x + \frac{450}{x}$

(b) $156 - 2x + 3(\frac{150}{x})$

(c) $156 - 2x - 15(\frac{3}{x})$

(d) $156 - 2x - 3(\frac{150}{x})$

iv) For what value of ‘x’, the printable area of the page is maximum?

(a) 15 cm

(b) 10 cm

(c) 12 cm

(d) 15 units

v) What should be dimension of the page so that it has maximum area to be printed?

(a) Length1cm, width15cm

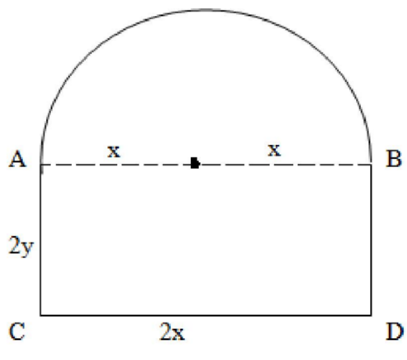
(b) Length15cm, width10cm

(c) Length15cm, width12cm

(d) Length150cm, width1cm

Case study 6.2

Mr Rajendra, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening. This window is having a perimeter of 10 m as shown below:



Based on the above information answer the following:

i) If 2x and 2y represents the length and breadth of the rectangular portion of the windows, then the relation between the variables is

(a) $4y - 2x = 10 - \pi$

(b) $4y = 10 - (2 - \pi)x$

(c) $4y = 10 - (2 + \pi)x$

(d) $4y - 2x = 10 + \pi$

ii) The combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x is

(a) $A = 10x + (2 + \frac{1}{2}\pi)x^2$

(b) $A = 10x - (2 + \frac{1}{2}\pi)x^2$

(c) $A = 10x - (2 - \frac{1}{2}\pi)x^2$

(d) $A = 4xy + \frac{1}{2}\pi x^2, where\ y = \frac{5}{2} + \frac{1}{4}(2 + \pi)x$

iii) The maximum value of area A, of the whole window is

(a) A= $\frac{50}{\pi - 4}\ m^2$

(b) A= $\frac{50}{\pi + 4}\ m^2$

(c) A= $\frac{100}{4 + \pi}\ m^2$

(d) A= $\frac{50}{4 - \pi}\ m^2$

iv) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible.

For this to happen, the length of rectangular portion of the window should be

(a) $\frac{20}{4+\pi}$

(b) $\frac{10}{4+\pi}$

(c) $\frac{4}{10+\pi}$

(d) $\frac{100}{4+\pi}$

v) In order to get the maximum light input through the whole window, the area (in sq. m) of only semi-circular opening of the window is

(a) $\frac{100\pi}{(4+\pi)^2}$

(b) $\frac{50\pi}{4+\pi}$

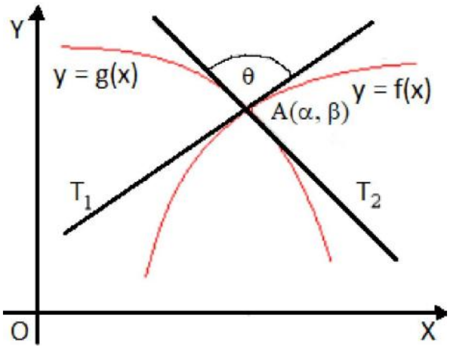
(c) $\frac{50\pi}{(4+\pi)^2}$

(d) Same as the area of rectangular portion of window.

Case study 6.3

Assuming that two ships follow the path of curves y=f(x) and y=g(x)

Let the two curves intersect each other at a point A(□,□).



When we draw tangents to these curves at the point of intersection, then 'the angle between these tangents' is called the 'angle between the two curves.

Using the information given above, answer the following with reference to the curves $y = x^2 \; and \; x = y^2$

(i) The points of intersection for the above curves are

(a) (0, 0), (1, ±1) (b) (0, 0), (1, 1)

(c) (0, −1), (1, 0) (d) (1, 0), (0, 1)

(ii) What are the numbers of points at which the given two curves intersect?

(a) 2 (b) 1 (c) 3 (d) 0

(iii) The slope of the tangent to the curve $x = y^2$ at the point of intersection of both the given curves, is

(a) $\frac{1}{2}, -\frac{1}{2}, 0$ (b) $\frac{1}{2}, 0$ (c) $-\frac{1}{2}, Not\ defined$ (d) $\frac{1}{2}, Not\ defined$

(iv) The slope of the tangent to the curve y= x^2 at the point of intersection of both the given curves, is

(a) 0, 2 (b) 2, -2 (c) 0, -1 (d) 2, -2, 0

(v) The angle of intersection of both the curves is

(a) $\pi, \tan^{-1}\frac{3}{4}$ (b) $\frac{\pi}{2}, \tan^{-1}\frac{4}{3}$ (c) $\frac{\pi}{2}, \tan^{-1}\frac{3}{4}$ (d) $0, \tan^{-1}\frac{3}{4}$

Case study 6.4

Neelam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of cardboard of side 18 cm. Based on the above information, answer the following questions:

(i) If x cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm, then x must lie in

(a) [0, 18] (b) (0,9)

(c) (0, 3) (d) None of these

(ii) Volume of the open box formed by folding up the cutting corner can be expressed as

(a) $V = x(18 - 2x)(18 - 2x)$ (b) $V = x(18 + x)(18 - x)$

(c) $V = \frac{1}{3}x(18+2x)(18 - 2x)$ (d) $V = x(18 - 2x)(18 - x)$

(iii) The values of x for which $\frac{dV}{dx} = 0$, are

(a) 3, 2 (b) 0, 3 (c) 0, 9 (d) 3, 9

(iv) Neelam is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(a) 13 cm (b) 8 cm (c) 3 cm (d) 2 cm

(v) The maximum value of the volume is

(a) 144 cm^3 (b) 232 cm^3 (c) 256 cm^3 (d) 432 cm^3

Case study 6.5

Peter's father wants to construct a rectangular garden with a rock wall on one side of the garden and wire fencing for the other three sides. He has 100 feet of wire fencing. Based on the above information, answer the following questions.

(i) To construct a garden such that the largest green carpet can be laid down in the garden, using the available 100 feet of fencing, we need to maximise its

(a) volume (b) area (c) perimeter (d) Height of the wall

(ii) If x denotes the length of side of garden perpendicular to rock wall and y denote the length of side parallel to rock wall, then find the relation representing total amount of fencing

(a) $x + 2y = 100$ (b) $x + 2y = 50$ (c) $y + 2x = 100$ (d) $y + 2x = 50$

(iii) Area of the garden as a function of x i.e., A(x) can be represented as

(a) $100 + 2x^2$ (b) $x - 2x^2$ (c) $100x - 2x^2$ (d) $100 - x^2$

(iv) Maximum value of A(x) occurs at x equals

(a) 25 feet (b) 30 feet (c) 26 feet (d) 31 feet

(v) Maximum area of garden will be

(a) 1200 sq. ft (b) 1000 sq. ft (c) 1250 sq. ft (d) 1500 sq. ft

Case study 6.6

The Government declare that farmers can get Rs. 200 per quintal for their potatoes on 1st February and after that, the price will be dropped by Rs. 2 per quintal per extra day. Ramu's father has 80 quintals of potatoes in the field and he estimates that crop is increasing at the rate of 1 quintal per day.

Based on the above information, answer the following questions

(i) If x is the number of days after 1st February, then price and quantity of potatoes respectively can be expressed as

(a) Rs. $(200 - 2x)$, $(80 + x)$ quintals

(b) Rs. $(200 - 2x)$, $(80 - x)$ quintals

(c) Rs. $(200 + x)$, 80 quintals

(d) None of these

ii) Revenue R as a function of x can be represented as

(a) $R(x) = 2x^2 - 40x - 16000$ (b) $R(x) = -2x^2 + 40x + 16000$

(c) $R(x) = 2x^2 + 40x - 16000$ (d) $R(x) = 2x^2 - 40x - 15000$

(iii) Find the number of days after 1st February, when Ramu's father attain maximum revenue.

(a) 10 (b) 20 (c) 12 (d) 22

(iv) On which day should Ramu's father harvest the potatoes to maximise his revenue?

(a) 11th February (b) 20th February

(c) 12th February (d) 22nd February

(v) Maximum revenue is equal to

(a) Rs. 16000 (b) Rs. 18000 (c) Rs. 16200 (d) Rs. 16500

Case study 6.7

An owner of a car rental company has determined that if they charge customers Rs. x per day to rent a car, where $50 \leq x \leq 200$, then number of cars (n), they rent per day can be shown by linear function

$$n(x) = 1000 - 5x.$$

If they charge Rs. 50 per day or less they will rent all their cars. If they charge Rs. 200 or more per day they will not rent any car.

Based on the above information, answer the following question.

(i) Total revenue R as a function of x can be represented as

(a) $1000x - 5x^2$ (b) $1000x + 5x^2$ (c) $1000 - 5x$ (d) $1000 - 5x^2$

(ii) If R(x) denote the revenue, then maximum value of R(x) occurs when x equals

(a) 10 (b) 100 (c) 1000 (d) 50

(iii) At x = 220, the revenue collected by the company is

(a) Rs. 10 (b) Rs. 500 (c) Rs. 0 (d) Rs. 1000

(iv) The number of cars rented per day, if x = 75 is

(a) 675 (b) 700 (c) 625 (d) 600

(v) Maximum revenue collected by company is

(a) Rs. 40000 (b) Rs. 45000 (c) Rs. 55000 (d) Rs. 50000

CASE STUDY 6.8

Mr. Vinay is the owner of apartment complex with 50 units. When he set rent at Rs. 8000/month, all apartments are rented. If he increases rent by Rs. 250/month, one fewer apartment is rented. The maintenance cost for each occupied unit is Rs. 500/month.

Based on the above information answer the following:

(i) If P is the rent price per apartment and N is the number of rented apartments, then profit is given by:

(a) NP (b) (N – 500). P (c) N. (P – 500) (d) none of these

(ii) If x represents the number of apartments which are not rented, then the profit expressed as a function of x is

(a) (50 – x) (30 + x) (b) (50 + x) (30 – x)

(c) 250(50 – x) (30 + x) (d) 250(50 + x) (30 – x)

(iii) If P = 8500, then N =

(a) 50 (b) 48 (c) 49 (d) 47

(iv) If P = 8250, then the profit is

(a) Rs. 379750 (b) Rs. 4,00,000 (c) Rs. 4,05,000 (d) Rs. 4,50,000

(v) The rent that maximizes the total amount of profit is

(a) Rs. 5000 (b) Rs. 10500 (c) Rs. 14800 (d) Rs. 14500

CASE STUDY 6.9

A concert is organised every year in the stadium that can hold 42000 spectators. With ticket price of Rs. 10, the average attendance has been 27000. Some financial expert estimated that price of a ticket should be determined by the function $p(x) = 19 - \frac{x}{3000}$ where x is the number of tickets sold. Based on the above information, answer the following questions.

(i) The revenue, R as a function of x can be represented as

(a) $19x - \frac{x^2}{3000}$ (b) $19 - \frac{x^2}{3000}$ (c) $19x - \frac{1}{3000}$ (d) none of these

(ii) The range of x is

(a) [27000, 42000] (b) [0, 27000]

(c) [0, 42000] (d) none of these

(iii) The value of x for which revenue is maximum, is

(a) 20000 (b) 27000 (c) 28500 (d) 28000

(iv) When the revenue is maximum, the price of the ticket is

(a) Rs. 8 (b) Rs. 5 (c) Rs. 9 (d) Rs. 9.5

(v) How many spectators should be present to maximize the revenue?

(a) 25000 (b) 27000 (c) 22000 (d) 28500

CASE STUDY 6.10

A can manufacturer designs a cylindrical can for a company making sanitizer and disinfector. The can is made to hold 5 litres of sanitizer or disinfector.

(i) If r cm be the radius and h cm be the height of the cylindrical can, then the surface area expressed as a function of r as

(a) $2\pi r^2$ (b) $2\pi r^2 + 5000$ (c) $2\pi r^2 + \frac{5000}{r}$ (d) $2\pi r^2 + \frac{10000}{r}$

(ii) The radius that will minimize the cost of the material to manufacture the can is

(a) $\left(\frac{500}{\pi}\right)^{1/3}$ (b) $\sqrt{\frac{500}{\pi}}$ (c) $\left(\frac{2500}{\pi}\right)^{1/3}$ (d) $\sqrt{\frac{2500}{\pi}}$

(iii) The height that will minimize the cost of the material to manufacture the can is

(a) $\left(\frac{2500}{\pi}\right)^{1/3} cm$ (b) $2\left(\frac{2500}{\pi}\right)^{1/3} cm$ (c) $\sqrt{\frac{2500}{\pi}}\, cm$ (d) $2\sqrt{\frac{2500}{\pi}}$

(iv) If the cost of material used to manufacture the can is Rs.100/m^2 and $\left(\frac{2500}{\pi}\right)^{1/3} = 9$, then the minimum cost is

(a) Rs. 16.7 (b) Rs. 18 (c) Rs. 19 (d) Rs. 20

(v) To minimize the cost of the material used to manufacture the can, we need to minimize the

(a) volume (b) curved surface area

(c) total surface area (d) surface area of the base

SOLUTION TO CASE STUDY 6.1

(i) (b) xy=150

Here the total length and width of the page is x and y, respectively (as seen in the figure)

Now we have, area of the page = xy

So, xy=150

(ii) (c) As the combined width of the margin at the top and bottom is 3 cm and the side 2 cm. Also, the total length and width of the page is x and y, respectively (as seen in the figure).

So, the area of printed page is, S=(x-3)(y-2)

(iii) (d) $156 - 2x - 3(\frac{150}{x})$

As the area of printed page is, S=(x-3)(y-2)=xy-2x-3y+6

From part (i) we have xy=150, so

S= 150-2x-3($\frac{150}{x}$)+6 $=156 - 2x - 3(\frac{150}{x})$

(iv) (a) 15 cm

from part (iii) we have

S $=156 - 2x - 3(\frac{150}{x})$

Which gives

$\frac{dS}{dx} = -2 + \frac{900}{x^2}$, Now

For local points of maxima and/or minima,

$\frac{dS}{dx} = 0 \; gives \; -2 + \frac{900}{x^2} = 0 \Rightarrow x^2 = 225 \Rightarrow \text{x} = \pm 15$

Here, $\frac{d^2S}{dx^2} = -\frac{1800}{x^3}$

Now $(\frac{d^2S}{dx^2})_{at\, x=15} = -ve$

Therefore, S is Maximum at x=15 cm

(v) (b) 10 cm

For maximum value of S, x= 15cm

So xy=150 gives y=10 cm

SOLUTION TO CASE STUDY 6.2

(i) (c) 4y=10-(п+2)x

Since perimeter of window=10

2x+4y+п x=10

4y=10-п x-2x

4y=10-(п+2)x

(ii) (b) Combined area $A = 10x - (2 + \frac{1}{2}\pi)x^2$

A= $x(4y) + \frac{1}{2}\pi x^2$

A= $x[10 - (\pi + 2)x] + \frac{1}{2}\pi x^2$

A= $10x - \pi x^2 - 2x^2 + \frac{1}{2}\pi x^2$

A= $10x - 2x^2 - \frac{1}{2}\pi x^2$

$$A = 10x - (2 + \frac{1}{2}\pi)x^2$$

(iii) (b) A= $\frac{50}{\pi+4}$

Here Combined area $A = 10x - \left(2 + \frac{1}{2}\pi\right)x^2$

$$\frac{dA}{dx} = 10 - (4 + \pi)x$$

For critical points,

$\frac{dA}{dx} = 0 \text{ which gives } 10 - (4 + \pi)x = 0 \Rightarrow x = \frac{10}{4+\pi}$

Now at $= \frac{10}{4+\pi}$, $\frac{d^2A}{dx^2} < 0$, so Combined area A is maximum when $x = \frac{10}{4+\pi}$

And Maximum Area $= 10x - \left(2 + \frac{1}{2}\pi\right)x^2$

$= 10\frac{10}{4+\pi} - \left(2 + \frac{1}{2}\pi\right)(\frac{10}{4+\pi})^2$

$= \frac{50}{4+\pi}$

(iv) (a) Length of rectangular portion of window $= 2(\frac{10}{4+\pi}) = \frac{20}{4+\pi}$ m

(v) (b) Area of semi-circular opening $= \frac{1}{2}\pi x^2 = \frac{1}{2}\pi(\frac{10}{4+\pi})^2 = \frac{50\pi}{(4+\pi)^2}\, m^2$

SOLUTION TO CASE STUDY 6.3

(i) (b) (0, 0), (1, 1)

We have $y = x^2 \text{ and } x = y^2$

On solving we get $x = (x^2)^2 \Rightarrow x = x^4 \Rightarrow x(x^3 - 1) = 0 \Rightarrow x(x - 1)(x^2 + x + 1) = 0$

$\Rightarrow x = 0 \text{ or } x = 1 \text{ as } (x^2 + x + 1) \neq 0$ for any real number

For $x = 0, y = 0$ and for $x = 1, y = 1$

So the point of intersection of the given curves are (0,0) and (1,1)

(ii) (b) 2

From part (i) It is clear that the given curves will intersect each other at 2 points only.

(iii) (d) $\frac{1}{2}, Not\ defined$

We have $x = y^2 \Rightarrow 1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

Now at (0,0) we have $\frac{dy}{dx} = \frac{1}{2y} = \frac{1}{0}$ $i.e. not\ defined$

Also, at (1,1) we have $\frac{dy}{dx} = \frac{1}{2}$

(iv) (a) 0, 2

We have $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

Now at (0,0) we have $\frac{dy}{dx} = 0$

Also, at (1,1) we have $\frac{dy}{dx} = 2$

(v) (c) $\frac{\pi}{2}, \tan^{-1}\frac{3}{4}$

Angle of intersection of both the curves at (0, 0) is $\pi/2$

And Angle of intersection of both the curves at (1, 1) is $\tan^{-1}\left(\frac{3}{4}\right)$

SOLUTION TO CASE STUDY 6.4

(i) (b): Since, sidCe of square is of length 18 cm, therefore x can not exceed ½(18), So x∈ (0, 9).

(ii) (a): Clearly, height of open box = x cm

Length of open box = 18 – 2x

and width of open box = 18 – 2x

Volume(V) of the open box=lbh=x. (18–2x). (18–2x)

(iii) (d): We have, V = x. $(18 - 2x)^2$

$\frac{dV}{dx}$ = 1. $(18 - 2x)^2$ +x.2(18 – 2x)(– 2)

$\frac{dV}{dx}$ = (18 – 2x). [(18 – 2x)–4x]

$\frac{dV}{dx}$ = (18 – 2x). (18 – 6x)

Now, $\frac{dV}{dx} = 0 \Rightarrow 18 - 2x = 0$ or $18 - 6x = 0$

$\Rightarrow x = 9$ or 3

(iv) We have, $V = x.(18 - 2x)^2$

And $\frac{dV}{dx} = (18 - 2x).(18 - 6x)$

So $\frac{d^2V}{dx^2} = (18 - 2x).(-6) + (18 - 6x)(-2)$

$= (-2).[54 - 6x + 18 - 6x]$

$= (-2).[72 - 12x] = 24x - 144$

For $x = 3$, $\frac{d^2V}{dx^2} < 0$

For $x = 9$, $\frac{d^2V}{dx^2} > 0$

So, volume will be maximum when $x = 3$.

(v) (d): We have, $V = x(18 - 2x)^2$, which will be maximum when $x = 3$.

Maximum volume $= 3(18 - 6)^2 = 3 \times 12 \times 12 = 432$ cm^3

SOLUTION TO CASE STUDY 6.5

(i) (b): Clearly, we need to maximize its area.

(ii) (c): Required relation is given by $2x + y = 100$.

(iii) (c): Area of garden as a function of x can be represented as

$A(x) = x \cdot y = x(100 - 2x) = 100x - 2x^2$

(iv) (a): $A(x) = 100x - 2x^2 \Rightarrow A'(x) = 100 - 4x$

For the area to be maximum'

$A'(x) = 0$

$\Rightarrow 100 - 4x = 0 \Rightarrow x = 25$ ft.

(v) (c): Maximum area of the garden $= 100(25) - 2(25)^2$

$= 2500 - 1250 = 1250$ sq. ft

SOLUTION TO CASE STUDY 6.6

(i) (a): Let x be the number of extra days after 1st February

Price =Rs. $(200 - 2.x)$ = Rs. $(200 - 2x)$

Quantity = 800 quintals + x (1 quintal per day)

= $(80+ x)$ quintals

(ii) (b): $R(x)$ = Quantity × Price

$= (80+ x).(200 - 2x)$

$= 16000 - 160x + 200x - 2x^2$

(iii) (a): We have, $R(x) = 16000 + 40x - 2x^2$

$\Rightarrow R'(x) = 40 - 4x \Rightarrow R''(x) = -4$

For R(x) to be maximum, $R'(x) = 0$ and $R''(x) < 0$

$\Rightarrow 40 - 4x = 0 \Rightarrow x = 10$

(iv) (a): Ramu's father will attain maximum revenue after 10 days.

So, he should harvest the potatoes after 10 days of 1st February i.e., on 11th February.

(v) (c): Maximum revenue is collected by Ramu's father when x = 10

Maximum revenue = R(10)

$= 16000 + 40(10) - 2(10)2$

$= 16000 + 400 - 200 = 16200$

SOLUTION TO CASE STUDY 6.7

17. (i) (a): Let x be the price charge per car per day and n be the number of cars rented per day.

$R(x) = n.x = (1000 - 5x).x = -5x^2 + 1000x$

(ii) (b): We have, $R(x) = 1000x - 5x^2 \Rightarrow R'(x)= 1000 - 10x$

For R(x) to be maximum or minimum, $R'(x)=0 \Rightarrow 1000 - 10x=0 \Rightarrow x=100$

Also, $R''(100) = -10 < 0$, Thus, R(x) is maximum at x = 100

(iii) (c): If company charge Rs. 200 or more, they will not rent any car. Then revenue collected by him will be zero.

(iv) (c): If x = 75, number of cars rented per day is given by

$n = 1000 - 5 \times 75 = 625$

(v) (d): At x = 100, R(x) is maximum.

Maximum revenue = $R(100) = -5(100)2 + 1000(100)$ =Rs. 50,000

SOLUTION TO CASE STUDY 6.8

(i) (c): If P is the rent price per apartment and N is the number of rented apartments, the profit is given by $P(N) = NP - 500\,N = N.\,(P - 500)$

[Since Rs. 500/month is the maintenance charges for each occupied unit]

(ii) (c): Now, if x be the number of non-rented apartments,

then $N = 50 - x$ and $P = 8000 + 250\,x$

Thus, $P = N.\,(P - 500) = (50 - x)\,(8000 + 250\,x - 500)$

= (50 – x) (7500 + 250 x) = 250(50 – x) (30 + x)

(iii) (b): Clearly, if P = 8500, then

8500 = 8000 + 250 x ⇒x = 2 ⇒N = 48

(iv) (a): Also, if P = 8250, then

8250 = 8000 + 250 x⇒ x = 1 and so profit

P (1) = 250(50 – 1) (30 + 1) =Rs. 379750

(v) We have P(x)=250(50 – x) (30 + x)

P′(x)= 250[(−1) (30 + x) + 250(50 – x) (1)]

P′(x)= 250[−30– x+50 – x]

P′(x)= 250[20– 2x]

For maxima/minima, put P′(x) = 0 ⇒ 20 – 2x = 0 ⇒ x = 10

Hence, the rent that maximizes the profit is Rs. 10500.

Thus, price per apartment is, P = 8000 + 2500 = Rs. 10500

Solution to Case study 6.9

(i) (a): Let p be the price per ticket and x be the number of tickets sold.

Then, revenue function R(x)=p.x = $(19-\frac{x}{3000}).x = 19x-\frac{x^2}{3000}$

(ii) (c): Since, more than 42000 tickets cannot be sold.So, range of x is [0, 42000].

(iii) (c): We have, R(x) =$19x-\frac{x^2}{3000}$, ⇒ R′(x) =$19-\frac{x}{1500}$

For maxima/minima, put R′(x) = 0 ⇒ $19-\frac{x}{1500}=0 \Rightarrow x=28500$

Also, R′′(x)=$-\frac{1}{1500}$ <0

(iv) (d): Maximum revenue will be at x = 28500

Price of a ticket =$19-\frac{x}{3000}=19-\frac{28500}{3000}\ 19-9.5=9.5$

(v) (d): Number of spectators will be equal to number of tickets sold when revenue is maximum. Required number of spectators = 28500

Solution to Case study 6.10

(i) (d): Given r cm is the radius and h cm be the height of required cylindrical can. And volume = 5 liters = 5000 cm^3

$$\pi r^2 h = 5000 \Rightarrow h=\frac{5000}{\pi r^2}$$

Now, the surface area, as a function of r is given by

S(r) $= 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r.\frac{5000}{\pi r^2}$ $=2\pi r^2 + \frac{10000}{r}$

(ii) (c): Now, S(r) $=2\pi r^2 + \frac{10000}{r} \Rightarrow$ S′(r) $=4\pi r - \frac{10000}{r^2}$

To find critical points, put S′(r) = 0 $\Rightarrow 4\pi r - \frac{10000}{r^2} = 0 \Rightarrow r^3 = \frac{10000}{4\pi}$

$\Rightarrow r = \left(\frac{2500}{\pi}\right)^{1/3}$, Also S′′(r)>0, Thus, the critical point is the point of minima.

(iii) (b): The cost of material for the can is minimized when $r = \left(\frac{2500}{\pi}\right)^{1/3}$ cm and the height is $h = \frac{5000}{\pi r^2} = \frac{5000}{\pi\left(\left(\frac{2500}{\pi}\right)^{1/3}\right)^2} = \frac{2(2500)}{\pi\left(\frac{2500}{\pi}\right)^{2/3}} = 2\left(\frac{2500}{\pi}\right)^{1/3}\ cm$

(iv)(a): We have, minimum surface area= S(r=$\left(\frac{2500}{\pi}\right)^{1/3}$)= $2\pi r^2 + \frac{10000}{r}$

$= \frac{2\pi r^3 + 10000}{r}$

$= \frac{2\pi\frac{2500}{\pi} + 10000}{\left(\frac{2500}{\pi}\right)^{1/3}} = \frac{15000}{\left(\frac{2500}{\pi}\right)^{1/3}} = \frac{15000}{9}$ = 1666.67 cm^2

Cost of 1 m^2 material = Rs. 100

Therefore, Cost of 1 cm^2 material $=\frac{1}{100}$

Minimum cost=1666.67($\frac{1}{100}$) =Rs. 16.66 = Rs. 16.7

(v) (c): To minimize the cost we need to minimize the total surface area.

BY RAJENDRA PARMAR, PGT (MATHS) KV VALSURA

CH- 7 : INTEGRALS

1 MARK QUESTIONS

1. Evaluate : $\int (ae)^x$dx, a > 0

2. Evaluate : $\int \frac{\cos x - \cos 2x}{1 - cosx}$ dx

3. Evaluate : $\int \frac{x^3 - x^2 + x - 1}{x^2 + 1}$ dx

4. Evaluate : $\int \frac{1}{\sqrt{9 - 25x^2}}$ dx

5. Evaluate : $\int \frac{1}{\sqrt{9 + 8x - x^2}}$ dx

2 MARKS QUESTIONS

6. Evaluate : $\int \frac{1}{cos^2x(1 - tanx)^2}$ dx

7. Evaluate : $\int \frac{\sin(x - a)}{sinx}$ dx

8. Evaluate : $\int_0^1 xe^x$dx

9 If $\int_1^a (3x^2$+2x+1)dx = 11, find real values of a.

10. Evaluate : $\int_0^1 \frac{1}{\sqrt{1+x} + \sqrt{x}}$dx

3 MARKS QUESTIONS

11. Evaluate : $\int tan^4$ x dx

12. Evaluate : $\int \frac{4x+1}{x^2+3x+2}$ dx

13. Evaluate : $\int \frac{\sqrt{tanx}}{sinxcosx}$ dx

14. Evaluate : $\int \frac{sinx-\cos x}{\sqrt{sin2x}}$ dx

15. Evaluate : $\int e^x(\frac{2+sin2x}{1+cos2x})$ dx

5 MARKS QUESTIONS

16. Evaluate : $\int_0^1 cot^{-1}(1 - x + x^2)$dx

17 Evaluate : $\int_0^1 \frac{\log(1+x)}{1+x^2}$dx

18 Evaluate : $\int_{-1}^{\frac{3}{2}} |xsin\pi x|$dx

19 Evaluate : $\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}$ dx

20 Evaluate : $\int \frac{1}{\cos(x-a)\cos(x-b)}$ dx

SOLUTIONS

1. $\int (ae)^x$dx = $\frac{(ae)^x}{\log(ae)}$ +c

2. $\int \frac{\cos x - \cos 2x}{1-cosx}$ dx

= $\int \frac{\cos x - \cos 2x}{1-cosx}$ dx

= $\int \frac{\cos x-(2cos^2-1)}{1-cosx}$ dx

= $\int \frac{-(2cos^2x-cosx-1)}{1-cosx}$

=$\int \frac{(2cosx+1)(cosx-1)}{cosx-1}$dx

=$\int (2cosx + 1)$dx

= 2sinx + 1+c

3. $\int \frac{x^3 -x^2 +x-1}{x^2+1}$ dx

= $\int \frac{(x^2 +1)(x-1)}{x^2+1}$dx

= $\int (x-1)$dx

= $\frac{x^2}{2}$ - x + c

4. $\int \frac{1}{\sqrt{9-25x^2}}$

= $\int \frac{1}{\sqrt{3^2-(5x)^2}}$

= $\frac{1}{5}$ $sin^{-1}(\frac{5x}{3})$ + c

5. $\int \frac{1}{\sqrt{9+8x-x^2}}$

= $\int \frac{1}{\sqrt{5^2-(x-4)^2}}$dx

= $sin^{-1}(\frac{x-4}{5})$ +c

6. $\int \frac{1}{cos^2x(1-tanx)^2}$ dx

= $\int \frac{sec^2x}{(1-tanx)^2}$ dx

= $\int \frac{dt}{(1-t)^2}$ where t= tanx

= $\frac{1}{1-t}$ +c = $\frac{1}{1-tanx}$ +c

7. $\int \frac{\sin(x-a)}{sinx}$dx

= $\int \frac{(\sin x\cos a - cosxsina)}{sinx}$dx

= xcosa - sina logsinx +c

8. $\int_0^1 xe^x$dx

=$[xe^x]_0^1$ - $\int_0^1 1.e^x$dx

= 1

9. $\int_1^a(3x^2$+2x+1)dx = 11

$\rightarrow$ $[x^3 + x^2 + x]_1^a$= 11

$\rightarrow a^3 + a^2 + a$ -14 =0

$\rightarrow$ a = 2

10. $\int_0^1 \frac{1}{\sqrt{1+x} + \sqrt{x}}$dx

= $\int_0^1 \frac{\sqrt{1+x} - \sqrt{x}}{1}$ dx

= $\frac{4}{3}$ ($\sqrt{2}$ - 1)

11. $\int tan^4$ x dx

= $\int tan^2$x(sec^{2}x – 1)dx

=$\int(tan^2$xsec^{2}x – sec^{2}x +1)dx

= $\frac{tan^3x}{3}$ –tanx +x +c

12. $\int \frac{4x+1}{x^2+3x+2}$ dx

$=\int \frac{2(2x+3)-5}{x^2+3x+2}$dx

$= \int \frac{2(2x+3)}{x^2+3x+2}$dx $-\int \frac{5}{x^2+3x+2}$dx

$= 2\log I\, x^2+3x+2\, I - 5\log I\frac{x+1}{x+2}I + C$

13 $\int \frac{\sqrt{tanx}}{sinxcosx}$dx

$= \int \frac{\text{tanx}}{\sqrt{tanx}sinxcosx}$dx

$=\int \frac{sec^2x}{\sqrt{tanx}}$ dx =

$2\sqrt{tanx}$ +c

14 sinx − cos x =t

→ sin2x = 1- t^2

→ $\int \frac{sinx - \cos x}{\sqrt{sin2x}}$dx

$= \int \frac{\text{dt}}{\sqrt{1-t^2}}$

$= sin^{-1}$(sinx −cosx)+ c

15 $I = \int e^x(\frac{2+sin2x}{1+cos2x})$dx

$= \int e^x(sec^2x + tanx)$dx

$= e^x tanx + c$

16 $\int_0^1 cot^{-1}(1 - x + x^2)$dx

$=\int_0^1 tan^{-1}(\frac{1}{1 - x + x2})$dx

$=\int_0^1 tan^{-1}(\frac{x+(1-x)}{1-x(1-x)})$dx

$=2\int_0^1 tan^{-1}x$dx

$= \frac{\pi}{2} - \log 2$

17 $\int_0^1 \frac{\log(1+x)}{1+x^2}$

$= \int_0^{\frac{\pi}{4}} \frac{\log(1+tan\theta)}{1+tan^2\theta} sec^2\theta d\theta$

$= \int_0^{\frac{\pi}{4}} \log(1+tan\theta)\ d\theta$

$= \frac{\pi}{8}\log 2$

18 $\int_{-1}^{\frac{3}{2}} |xsin\pi x| dx = \int_{-1}^{1} |xsin\pi x| dx + \int_{1}^{\frac{3}{2}} |xsin\pi x| dx$

$= \int_{-1}^{1} |xsin\pi x| dx + \int_{1}^{\frac{3}{2}} |xsin\pi x| dx$

$= \int_{-1}^{1} xsinx\ dx - \int_{-1}^{\frac{3}{2}} xsinx\ dx$

$= 2\int_{0}^{1} xsinx\ dx - \int_{-1}^{\frac{3}{2}} xsinx\ dx$

$= \frac{3\pi+1}{\pi^2}$ (using integration by parts)

19 $\frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}$

$= \frac{(y+1)(y+4)}{(y+3)(y-5)}$

$= \frac{y^2+5y+4}{y^2-2y-15}$ ($y = x^2$)

$= 1+ \frac{7y+9}{(y+3)(y-5)}$

$= 1+ \frac{1}{4(y+3)} + \frac{27}{8(y-5)}$

$= 1+ \frac{1}{4(x^2+3)} + \frac{27}{8(x^2-5)}$

$\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)} = \int (1 + \frac{1}{4(x^2+3)} + \frac{27}{8(x^2-5)})\ dx$

$= x + \frac{1}{4\sqrt{3}}\ tan^{-1}\frac{x}{\sqrt{3}} + \frac{27}{8\sqrt{5}} \log\frac{x-\sqrt{5}}{x+\sqrt{5}} + c$

20 $\int \frac{1}{\cos(x-a)\cos(x-b)}$dx

= $\frac{1}{\sin(a-b)}\int \frac{\sin((x-b)-(x-a))}{\cos(x-a)\cos(x-b)}$dx

= $\frac{1}{\sin(a-b)}$log $\frac{\cos(x-a)}{\cos(x-b)}$ +c

CASE STUDY QUESTION

21 For any definite integral $\int_0^a f(x)dx$

(i) Which of the following is correct ?

(a)$\int_0^a f(x)dx$= $\int_0^a f(a-x)dx$

(b) $\int_0^a f(x)dx$= $\int_0^a f(2x)dx$

(c) $\int_0^a f(x)dx$= $\int_0^a f(a+x)dx$

(d) None of these

(ii) If I1 = $\int_0^{\frac{\pi}{2}} logsinxdx$ and I2 = $\int_0^{\frac{\pi}{2}} logcosxdx$,then

(a) I1< I2

(b) I1> I2

(c) I1=I2

(d) None of these

(iii) $\int_0^{\frac{\pi}{2}} logsinxdx$ =

(a) $\frac{\pi}{2}$ log 2

(b) $-\frac{\pi}{2}$ log 2

(c) πlog 2

(d) 2log2

(iv) If f(2a-x) = - f(x) then $\int_0^{2a} f(x)dx$ =

(a) 1

(b) 0

(c) $2\int_0^{a} f(x)dx$

(d) None of these

(v) $\int_0^{\frac{\pi}{2}} logtanxdx$=

(a) -1

(b) 1

(c) 2

(d)0

SOLUTIONS : (i) a (ii) c (iii) b (iv) b (v) d

BY A. P. SRIVASTAVA, PGT(MATHS) KV AFS, BARODA

CH-8

APPLICATION OF INTEGRAL

Questions for High achiever (3 or 5 marks questions)

1. Find the area of the region bounded by the curve y = x^2 and the line y = 4.
2. Find the area enclosed by the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
3. Find the area of the region in the first quadrant enclosed by the x-axis, the line y =x, and the circle: $x^2 + y^2 = 32$.
4. Find the area of the region bounded by the curve y^2 = x and the lines x = 1, x= 4 and the x-axis in first quadrant.
5. Find the area of the region bounded by y^2 = 9x, x= 2, x= 4 and the x-axis in the first quadrant.
6. Find the area of the region in the first quadrant enclosed by x-axis, line x = $\sqrt{3}$ y and the circle $x^2 + y^2 = 4$
7. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line = $\frac{a}{\sqrt{2}}$.
8. The area between x = y^2 and x= 4 is divided into two equal parts by the line x= a, find the value of a.
9. Find the area of the region bounded by the parabola y =x^2 and y= l x l.
10. Find the area bounded by the curve x^2 = 4y and the line x= 4y– 2.
11. Find the area of the region bounded by the curve y^2 = 4x and the line x = 3.
12. Using integration find the area of region bounded by the triangle whose vertices are (1, 0), (2, 2) and (3, 1).
13. Using integration find the area of region bounded by the triangle whose vertices are (– 1, 0), (1, 3) and (3, 2).
14. Using integration find the area of the triangular region whose sides have the equations y= 2x+ 1, y= 3x+ 1 and x= 4.
15. Using the method of integration find the area bounded by the curve lxl +l yl = 1
16. Draw the graph of $y = |x+1|$ and using integration find the area below $y = |x+1|$above x axis and between x = - 4 to x =2.
17. Using method of integration, find the area of the region bounded by the lines 3x - y - 3 =0, 2x + y - 12 = 0 and x - 2y - 1=0.

Answers:

1. 32/3 sq. unit
2. $\pi ab\ sq.unit$
3. $4\pi\ sq.unit$
4. 14/3 $sq.unit$
5. $16 - 4\sqrt{2}\ sq.unit$
6. $\pi/3\ sq.unit$
7. $\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$ sq. unit
8. 1/3 unit square
9. $\frac{1}{3}$unit squares
10. $\frac{9}{8}$unit squares X
11. 8 $\sqrt{3}$ sq. unit
12. 3/2 sq. unit
13. 4 sq. unit
14. 8 sq. unit
15. 2 sq. unit
16. 9 sq. Units
17. 10 sq. units

BY MS BHAVNA SUTARIA, PGT(MATHS) KV CRPF, GANDHINAGAR

CH-9 . DIFFERENTIAL EQUATIONS

1. Write the sum of order and degree of the differential equations: $\frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}} = c$

Solution: $\frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}} = c$

$$\Rightarrow \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2} = c\frac{d^2y}{dx^2} \Rightarrow \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^3 = c^2\left(\frac{d^2y}{dx^2}\right)^2$$

$\therefore\ Order = 2 \ \ and\ \ Degree = 2$

$Sum = 2 + 2 = 4$

2. The order of the differential equation represented by the family of curves $y = a\,sin(bx + c)$, $where\ \boldsymbol{a\ and\ c}\ being\ parameter$.

Solution: $Order = 2$ $\boldsymbol{as\ it\ has\ two\ parameters}$.

3. Verify that $y = x\sin x$ is solution of differential equation $x\,y' = y + x\sqrt{x^2 - y^2}$.

Solution: $y = x\sin x$ ……… (1)

Differentiating w. r. t. x, we get

$y' = \sin x + x\cos x$

$\Rightarrow\ x\,y' = x\sin x + x^2\cos x \quad \Rightarrow\ x\,y' = y + x^2\sqrt{1 - sin^2x}$

$\Rightarrow\ x\,y' = y + x^2\sqrt{1 - \frac{y^2}{x^2}}$, $\quad using\ equation\ (1)$

$\Rightarrow\ x\,y' = y + x\sqrt{x^2 - y^2}$

4. Solve the differential equation $(x + 2)\frac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$

Solution: $(x + 2)\frac{dy}{dx} = x^2 + 4x - 9$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+4x-9}{x + 2} \quad \Rightarrow \frac{dy}{dx} = x + 2 - \frac{13}{x + 2}$$

$$\Rightarrow \int dy = \int\left\{x + 2 - \frac{13}{x + 2}\right\}dx$$

$$\Rightarrow\ y = \frac{x^2}{2} + 2x - 13\log|x + 2| + \boldsymbol{C}$$

5. Solve the differential equation: $\left(x\, e^{y/x} + y\right) dx = x\, dy$, $y(1) = 1$

Solution: $\left(x\, e^{y/x} + y\right) dx = x\, dy$

$$\Rightarrow \frac{dy}{dx} = e^{y/x} + \frac{y}{x}$$

This is a homogeneous differential equation.

$Put\ y = vx \ \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$\Rightarrow v + x\frac{dv}{dx} = e^{v} + v \quad \Rightarrow e^{-v}\, dv = \frac{dx}{x}$

On integration, we get

$$\int e^{-v}\, dv = \int \frac{dx}{x}$$

$$\Rightarrow -e^{-v} = \log|x| + \boldsymbol{C}$$

$$\Rightarrow -e^{-y/x} = \log|x| + \boldsymbol{C} \quad \ldots\ldots\ldots(\mathbf{1})$$

It is given that $y(1) = 1$, so we get

$$-e^{-1} = \boldsymbol{C}$$

$$\Rightarrow \boldsymbol{C} = -\frac{\mathbf{1}}{\boldsymbol{e}}$$

Putting this value in equation (1), we get

$$e^{-y/x} = \frac{\mathbf{1}}{\boldsymbol{e}} - \log|x|$$

$$\Rightarrow -\frac{y}{x} = log\{1 - e\, log|x|\} - 1$$

$$y = x - x\ log\{1 - e\, log|x|\}$$

6. Solve the differential equation: $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ when $y(0) = 0$.

Solution: The given differential equation can be written as

$$\frac{dy}{dx} + \frac{2\,x}{1 + x^2}\, y = \frac{4\,x^2}{1 + x^2}$$

It is linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where

$$P = \frac{2\,x}{1 + x^2} \ \ \boldsymbol{and}\ \ Q = \frac{4\,x^2}{1 + x^2}$$

$$\boldsymbol{I.F.} = e^{\int P\, dx} = e^{\int \frac{2\,x}{1 + x^2} dx} = e^{log\left(1 + x^2\right)} = 1 + x^2$$

Solution is $y\,.(\boldsymbol{I.F.}) = \int Q.(\boldsymbol{I.F.})\, dx + \boldsymbol{C}$

$$y\,.(1 + x^2) = \int 4\,x^2\, dx + \boldsymbol{C}$$

$y.(1 + x^2) = \frac{4\ x^3}{3} + \boldsymbol{C} \ldots\ldots\ldots (\mathbf{1})$

It is given that $y(0) = 0$, so we get

$\boldsymbol{C} = \mathbf{0}$

Putting this value in equation (1), we get

$y. = \frac{4\ x^3}{3\ (1 + x^2)}$

1 MARK QUESTIONS

1. Write the order and degree of the following differential equations:

a) $\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$

b) $\frac{d^4y}{dx^4} + \sin\left(\frac{d^3y}{dx^3}\right) = 0$

c) $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

d) $\frac{dy}{dx} + y = e^x$.

e) $\sqrt{1 + \frac{dy}{dx}} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$

f) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + siny = 0$

g) $\frac{d^4y}{dx^4} + \log\left(\frac{d^3y}{dx^3}\right) = 0$

h)

2. Write the sum of order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y^5 = 0$

3. Find the product of the order and degree of differential equation $x\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$.

4. What is the order of the differential equation of the family of circles $x^2 + y^2 = a^2$. 'a' is an arbitrary constant?

5. Write the number of arbitrary constants in the general solution of a differential equation of fourth order.

6. How many arbitrary constants are there in the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$ $when\ y(0) = 1$.

7. Write the integrating factor of the differential equation.

a) $\frac{dy}{dx} + 2y = e^{3x}$

b) $\frac{dy}{dx} - \frac{1}{x}y = 2x^2$

c) $\sqrt{x}\frac{dy}{dx} + y = e^{-2\sqrt{x}}$.

d) $sinx\frac{dy}{dx} + y\ cosx = cosx.sin^2x$

e) $xlogx\frac{dy}{dx} + y = 2logx$

f) $\frac{dy}{dx} + y = \frac{1+y}{x}$

8. For what value of n is the following a homogeneous differential equation: $\frac{dy}{dx} = \frac{x^3 - y^n}{x^2 y + x y^2}$

9. Write the number of arbitrary constants in the particular solution of a differential equation of third order.

10. Write the number of arbitrary constants in a particular solution of the differential equation $tan\, x\, dx + tan\, y\, dy = 0$.

2 MARK QUESTIONS

11. Solve the differential equations.

a) $\frac{dy}{dx} = 2^{-y}$

b) $\frac{dy}{dx} = x^3 e^{-2y}$

c) $\frac{dy}{dx} = \frac{x+1}{2-y}$,

d) $\log\left(\frac{dy}{dx}\right) = 3x + 4y$

e) $\frac{dy}{dx} = e^{y+x} + e^y x^2$

f) $\frac{dy}{dx} = \frac{xe^x logx + e^x}{xcosy}$

12. Solve the differential equations

a) $\frac{dy}{dx} - \frac{1}{x}.y = 2x^2$

b) $\frac{dy}{dx} + secx.y = tanx$

3 MARK QUESTIONS

13. Solve the following differential Equations:

a) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, given that y(0) = √3

b) $\frac{dy}{dx} = x^3 cosecy$, y(0) = 1.

c) $\frac{dy}{dx} = ytanx$, y = 1 when x = 0.

d) $xy\frac{dy}{dx} = (x+2)(y+2)$, passes through (1, –1)

14. Solve the following differential equations:

a) $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

b) $(x^3 + y^3)dy - x^2 y dx = 0$

c) $(3xy + y^2)\, dx + (x^2 + xy)\, dy = 0\, ;\ x = 1,\ y = 1$

d) $x\frac{dy}{dx} = y - x.\tan\left(\frac{y}{x}\right) = 0$

15. Solve the following differential equations

a) $\frac{dy}{dx} + y = cosx - sinx$

b) $cosx\frac{dy}{dx} + y = sinx, y\,(0) = 2$

c) $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$

d) $(1+x^2)dy+2xydx=cotx.dx, x\neq 0$

e) $xdy+(y-x^3)dx=0$

f) $xdy-(y+2x^2)dx=0$

g) $(\tan^{-1}x-y)dx=(1+x^2)dy$

CASE – STUDY BASED QUESTION:

16. A helicopter moves on a path in such a way that at any point $(x,\ y)$ of the path the slope of tangent is twice the slope of the line – segment joining the point of contact to the point $(-4,-3)$.

a) The differential equation according to the given condition, is:

(i) $\frac{dy}{dx}=2\left(\frac{y+3}{x+4}\right)$ **(iii)** $\frac{dy}{dx}=3\left(\frac{y+2}{x+4}\right)$

(ii) $\frac{dy}{dx}=2\left(\frac{y+4}{x+3}\right)$ **(iv)** $\frac{dy}{dx}=4\left(\frac{y+3}{x+2}\right)$

b) The solution of the differential equation of part **a),** is:

(i) $(y+3)=\boldsymbol{C}(x+4)^2$ **(iii)** $(y+2)=\boldsymbol{C}(x+4)^2$

(ii) $(y+4)=\boldsymbol{C}(x+3)^2$ **(iv)** $(y+3)=\boldsymbol{C}(x+2)^2$

c) If the helicopter passes through the point $(-2,\ 1)$, then the equation of the path is:

(i) $(y+3)=(x+4)^2$ **(iii)** $(y+2)=(x+4)^2$

(ii) $(y+4)=(x+3)^2$ **(iv)** $(y+3)=(x+2)^2$

d) The order and degree of the differential equation of part (**a),** is:

(i) $Order=1,\ Degree=1$ **(iii)** $Order=1,\ Degree=2$

(ii) $Order=2,\ Degree=1$ **(iv)** $Order=2,\ Degree=\mathbf{2}$

e) The name of the curve on which helicopter is moving, is:

(i) Circle

(ii) Parabola

(iii) Ellipse

(iv) Straight line

BY MR ASHUTOSH RAI, PGT (MATHS)
KV ONGC, CHANDKHEDA

CH-10 VECTOR ALGEBRA

1 MARKS

1 For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

2 Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ.

3 Find the position vector of the mid point of the vector joining the points P(2, 3, 4) and Q(4, 1, –2).

4 Find position vectors of the points which divides the join of the points 2a - 3b and 3a - 2b externally in the ratio 2:3.

5 Find the area of the parallelogram whose diagonals are $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-3\hat{i} + 4\hat{j} - \hat{k}$.

6 Find the value of **'x'** for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

7 If a vector makes angles **α, β, γ** with x-axis, y-axis, z-axis respectively, then what is the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

8 If the vector $a = 2\hat{i} - 3\hat{j}$ and $b = -6\hat{i} + m\hat{j}$ are collinear , find the value of m.

2 MARKS

9 If a unit vector a makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find **'θ'** and hence the components of a .

10 If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

11 If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$. But the converse need not be true. Justify your answer with an example.

12 Given that $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$. What can you conclude about the vectors $\vec{a}$ and $\vec{b}$?

13 Write all the unit vectors in XY-plane.

14 A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

3 MARKS

15 If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

16 Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$.

17 The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

18 Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

19 Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ externally in the ratio 1 : 2. Also, show that P is the mid point of the line segment RQ.

20 Show that the angle between two diagonals off a cube is $\cos^{-1}(1/3)$.

BY SEEMA SUROLIA, PGT- MATHS, KV AHMEDABAD CANTT

CH-11: 3 - DIMENSIONAL GEOMETRY

1-MARK QUESTIONS

Q-1 Find the distance of plane $\vec{r}\cdot\left(\hat{\imath}-\hat{\jmath}+2\hat{k}\right)=5$ from origin.

Hint/soln Convert the plane into normal form $\vec{r}\cdot(\hat{n})=p$ where p will give the distance of the plane from origin

Q-2 Write the equation of the plane parallel to XOY plane and passing through the point (1,4, -3)

Hint/soln Here $\vec{n}=(0,0,1)\ and\ \vec{a}=(1,4,-3)$ now use vector equation of the plane

Q-3 Find the intercept cut-off by the plane $3x-5y+z=-7$ on the Y-Axis.

Hint/soln Convert the equation of the plane to the Intercept form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Q-4 Write the direction cosines of the line $\frac{x-1}{3}=\frac{5-2y}{1}, z=3$

Hint/soln Convert the equation into standard form. Find the direction ratios of the vector parallel to the line and then convert it into unit vector to provide the DCs of the line.

Q-5 Find the equation of the line parallel to the Z-Axis and passing through the point with P.V. (1, -2,5)

Hint/soln $\vec{a}=(1,-2,5) and\ \vec{b}=(0,0,1)$. Now use the vector equation of the line.

2-MARKS QUESTIONS

Q-1 Find the equation of the line perpendicular to the plane $x-2y+3z=4$ which also passes through the point (1, -2, -3).

Hint/soln Normal of the plane will become vector b for the required line. Moreover, P.V. of the point on the line is already given. Now use the vector equation of the line.

Q-2 Check if the line $\frac{x-1}{3}=\frac{5-2y}{1}=\frac{z+1}{-1}$ intersects the plane $x-2y-3z=5$ Or not.

Hint/soln If $\vec{b} \cdot \vec{n} \neq 0$ then the line intersects the plane. (Provided that the line doesn't lie inside the plane)

Q-3 Show that the lines $\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{2y-8}{2} = \frac{z-5}{3}$ are coplanar.

Hint/soln Coplanar lines mean either parallel or intersecting lines.

- Check if $(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0\ 0r\ not$ if it is zero then lines are coplanar.

Q-4 Find the foot of perpendicular from (1,3,4) to the plane 2x-y+z+3=0

Hint/soln Find equation of the line passing through given point and perpendicular to the plane. Now find the point of intersection of this line and the plane. It is the foot of perpendicular.

Q-5 A variable plane moves so that the sum of the reciprocals of its intercepts on the coordinate axis is ½. Find the fixed point from which the plane is bound to pass.

Hint/soln Here 1/a + 1/b + 1/c = ½-------------------------(1)
Let the equation be x/a + y/b + z/c = 1
X= 2, y=2 and z=2 satisfies the above eqn. so the fixed point is (2,2,2)

3- MARKS QUESTIONS.

Q-1 Find the distance of the point P(2,4, -1) from the line
$$\frac{x+5}{7} = \frac{y+3}{4} = \frac{z-6}{-9}$$

Hint/soln Take general point Q (7t-5, 4t-3, -9t+6) on the given line. Form vector PQ. Scalar product of vector PQ with (7,4, -9) is zero so find t and hence Q. Now using distance formula find distance PQ.

Q-2 Find the shortest distance between the lines
$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}\ and\ \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Hint/soln Shortest distance between two skew lines is
$$\left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$$

Q-3 Find the distance of the point P(1, -5,9) from the plane x-y+z=5 measured along the line x=y=z

Hint/soln Line x=y=z can be written as $\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-0}{1}------(1)$

Now equation of line parallel to above line and passing through (1,-5,9) is $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}$ ------------------(2)

Now find point of intersection Q of line (2) with plane x-y+z=5

Then find the distance PQ by distance formula

Q-4 Verify if the plane x-5y+2z=0 contains the line

$$\frac{x-5}{3}=y=2+z \text{ or not}$$

Hint/soln Here the given line is $\frac{x-5}{3}=\frac{y-0}{1}=\frac{z-(-2)}{1}$

For the given plane the normal vector is $\vec{n}=(1,-5,2)$

For the given line $\vec{b}=(3,1,1)$

Here $\vec{b}\cdot\vec{n}$=0

but the point (5,0, -2) does not satisfy the plane. So, the line doesn't lie inside the given plane.

Q-5 Find the reflection of the point P(0,0,0) in the plane 3x+4y-6z+1=0

Hint/soln Find the foot of perpendicular Q from P to the plane.

Let point R be the reflection of P

Now use the fact that Q is the mid-point of PR. And get R.

5-MARKS QUESTIONS

Q-1 Find the coordinates of foot of perpendicular and the perpendicular distance of the point P (3,2,1) from the plane 2x-y+z=-1. Also find the image of the point in the plane.

Hint/soln Let Q be the foot of perpendicular from P to the given plane

Now equation of line PQ will be

$\frac{x-3}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$=t

So general point Q on line is (2t+3,-t+2,t+1)

Which lies on 2x-y+z=-1 so Substituting Q in plane get t and hence get Q. Which is the foot of perpendicular.

Let R be the image of P

Now use the fact that Q is the mid-point of PR, get R.

Use distance formula to find the distance PQ.

Q-2 Find the equation of line of intersection of the planes X+2y-z=1 and 2x+2y-3z=5

Hint/soln Take z=0 in both planes we get

X+2y=1 and 2x+2y = 5

Solving them we get x=4 and y = -3/2

For the required line $\vec{a}=(4,-3/2,0)$

Now calculate $\overrightarrow{n_1} \times \overrightarrow{n_2} = \vec{b}$

So equation of required line is as per $\vec{r} = \vec{a} + t\,\vec{b}$ where t is any real number.

Q-3 Find the vector equation of the plane containing the lines $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{5}$ and $\frac{x-3}{3} = \frac{y-3}{-2} = \frac{z-2}{5}$

Hint/soln For the required plane $\vec{a} = (2,1,-3)$

Moreover , for the required plane $\vec{n} = \overrightarrow{b_1} \times \overrightarrow{b_2}$

Now use the vector form of plane to find the required equation

Q-4 If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, find the value of k. also find the equation of the plane containing them.

Hint/soln General point on first line is P(2t+1,3t-1,4t+1)

General point on second line Q(s+3,2s+k,s)

Compare first and third coordinates of P and Q to get two linear equations of t and s. Solve them to get values of t and s. now compare second coordinates of P and Q to get value of k.

As in question number 3 above, find the equation of the plane containing these two intersecting lines.

Q-5 Find the equation of line passing through the point (-4,3,1) parallel to the plane x+2y-z=0 and perpendicular to the line

$$\frac{x+1}{3} = \frac{y-3}{-2} = \frac{z-2}{-1}$$

Hint/soln For the required line $\vec{a} = (-4,3,1)$

For the line $\vec{b} = \overrightarrow{b_1} \times \vec{n}$ where $\overrightarrow{b_1}$=(3,-2,-1), $\vec{n} = (1,2,-1)$

Now use vector equation of the line to find the vector equation.

BY SHIRINKUMAR J. PANDYA, PGT (MATHS) KV KRIBHCO SURAT

CH 12 (LINEAR PROGRAMMING)

OBJECTIVE QUESTIONS (1 ×12=12)

1. For the constraint of a linear optimizing function $z=x1+x2$, given by

$X1+x2\leq1$, $3x1+x2\geq3$ and $x1, x2\geq0$

A) There are two feasible regions

B) There are infinite feasible regions

C) There is no feasible region

D) None of these

2. Which of the following is not a vertex of the positive region bounded by the inequalities

$2x+3y\leq6$, $5x+3y\leq15$ and $x, y\geq0$

A) (0, 2)

B) (0, 0)

C) (3, 0)

D) None of these

3. For the constraints of a L.P. problem given by

$x1+x2\leq2000$, $x1+x2\leq1500$, $x2\leq600$ and $x1, x2\geq0$

, which one of the following points does not lie in the positive bounded region

A) (1000, 0)

B) (0, 500)

C) (2, 0)

D) (2000, 0)

4. A basic solution is called non-degenerate, if

A) All the basic variables are zero

B) None of the basic variables is zero

C) At least one of the basic variables is zero

D) None of these

5. The graph of $x \leq 2$ and $y \geq 2$ will be situated in the

A) First and second quadrant

B) Second and third quadrant

C) First and third quadrant

D) Third and fourth quadrant

6. A vertex of the linear inequalities

$2x+3y \leq 6$, $x+4y \leq 4$ and $x, y \geq 0$, is

A) (1, 0)

B) (1, 1)

C) (125, 25)

D) (25, 125)

7. In which quadrant, the bounded region for in equations

$x+y \leq 1$ and $x-y \leq 1$ is situated

A) I, II

B) I, III

C) II, III

D) All the four quadrants

8. The necessary condition for third quadrant region in xy-plane, is

A) $x>0, y<0$

B) $x<0, y<0$

C) $x<0, y>0$

D) $x<0$, $y=0$

9. For the following feasible region, the linear constraints are

A) $x\geq0$, $y\geq0$, $3x+2y\geq12$, $x+3y\geq11$

B) $x\geq0$, $y\geq0$, $3x+2y\leq12$, $x+3y\geq11$

C) $x\geq0$, $y\geq0$, $3x+2y\leq12$, $x+3y\leq11$

D) None of these

10. The value of objective function is maximum under linear constraints

A) At the center of feasible region

B) At (0, 0)

C) At any vertex of feasible region

D) The vertex which is at maximum distance from (0, 0)

11. Shaded region is represented by

A) $2x+5y\geq80$, $x+y\leq20$, $x\geq0$, $y\leq0$

B) $2x+5y\geq80$, $x+y\geq20$, $x\geq0$, $y\geq0$

C) $2x+5y\leq80$, $x+y\leq20$, $x\geq0$, $y\geq0$

D) $2x+5y\leq80$, $x+y\leq20$, $x\leq0$, $y\leq0$

12. Corner points of the feasible region determined by the system of linear

constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px+qy$, where p, q > 0. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is

(A) $p = 2q$

(B) $p = q/2$

(C) $p = 3q$

(D) $p = q$

2 MARKS QUESTIONS

13. A wholesale merchant wants to start the business of cereal with Rs. 24000. Wheat is Rs. 400 per quintal and rice is Rs. 600 per quintal. He has capacity to store 200 quintal cereal. He earns the profit Rs. 25 per quintal on wheat and Rs. 40 per quintal on rice. If he stores x quintal rice and y quintal wheat, then for maximum profit Find the objective function .

14. Mohan wants to invest the total amount of Rs. 15,000 in saving certificates and national saving bonds. According to rules, he has to invest at least Rs. 2000 in saving certificates and Rs. 2500 in national saving bonds. The interest rate is 8% on saving certificate and 10% on national saving bonds per annum. He invest Rs. x in saving certificates and Rs. y in national saving bonds. Then find the objective function for this problem .

15 (i). A firm produces two types of products A and B. The profit on both is Rs. 2 per item. Every product requires processing on machines M1 and M2 . For A, machines M1 and M2 takes 1 minute and 2 minute respectively and for B, machines M1 and M2 takes the time 1 minute each. The machines M1, M2 are not available more than 8 hours and 10 hours, any of day, respectively. If the products made x of A and y of B, Find the linear constraints for the L.P.P. except $x \geq 0$, $y \geq 0$.

15 (ii). Find the objective function for the above condition.

3 MARKS QUESTIONS

16 (i).In a test of Mathematics, there are two types of questions to be answered? short answered and long answered. The relevant data is given below

Type of questions

Time taken to solve

Marks

Number of questions

Short answered questions

5 minute

3

10

Long answered questions

10 minute

5

14

The total marks is 100. Students can solve all the questions. To secure maximum marks, a student solves x short answered and y long answered questions in three hours, then find the linear constraints except $x \geq 0$, $y \geq 0$.

16 (ii). Find the objective function for the above question .

16 (iii). Find the vertices of a feasible region of the above question .

16 (iv).Find the maximum value of objective function in the above question .

17. A manufacturing company makes two types of television sets; one is black and white and the other is color. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a colored set. The company can spend not more than Rs 648000 a week to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per colored set, how many sets of each type should be produced so that the company has maximum profit? Formulate this problem as a LPP given that the objective is to maximize the profit.

18. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP so that the manufacturer can maximise his profit.

(ii) How many of circuits of Type A and of Type B, should be produced by the manufacturer so as to maximise his profit?

(iii)Determine the maximum profit.

5 MARKS QUESTIONS

19. Solve the following linear programming problem graphically :

Maximize Z = x + y

subject to constraints; $x + 4y \leq 8$, $2x + 3y \leq 12$, $3x + y \leq 9$,

$x \geq 0$, $y \geq 0$.

20. Fill in the blanks in each of the (1 ×5=5).

(i). In a LPP, the linear inequalities or restrictions on the variables are called

__________.

(ii). In a LPP, the objective function is always __________

(iii). A feasible region of a system of linear inequalities is said to be __________ if it can be enclosed within a circle.

(iv). A corner point of a feasible region is a point in the region which is the __________ of two boundary lines.

(v). The feasible region for an LPP is always a __________ polygon.

21. State whether the statements are True or False. (1 ×5=5)

(i). If the feasible region for a LPP is unbounded, maximum or minimum of the objective function Z = ax + by may or may not exist.

(ii). Maximum value of the objective function Z = ax + by in a LPP always occurs at only one corner point of the feasible region.

(iii). In a LPP, the minimum value of the objective function Z = ax + by is always 0 if origin is one of the corner point of the feasible region.

(iv). In a LPP, the maximum value of the objective function Z = ax + by is always finite.

(V). Maximum value of Z = 11x + 7y subject to the constraints :

2x + y ≤ 6, x ≤ 2, x ≥0, y ≥ 0 is 42 .

HINTS & SOLUTIONS

1. C No feasible region

2. D None

3. D (2000,0)

4. B None of the basic variables is zero

5. A First & Second quadrant

6. C (12/5, 2/5)

7. D All the four quadrants

8. B x <0 , y <0

9. A x ≥0, y ≥0 , 3x + 2y ≥12, x + 3y ≥11

10. D The vertex which is at maximum distance from (0,0)

11. C 2x +5y≤ 80 , x +y≤ 20 , x ≥0 , y ≥ 0

12. B p = q/2

13. Max Profit z = 40x +25y

14. 0.08x +0.1 y

15. (i) x + y ≤8×60=480 & 2x + y ≤10 ×60= 600

(ii) 2x +2y

16. (i) 5x + 10y ≤ 180 , x ≤10 , y ≤ 14

(ii) 3x + 5y

(iii) Vertices (8,14),(10,13),(10,0) and (0,14)

(iv) Max z = 3(10) + 5(13) = 95

17. X + y ≤ 300 and 1800 x + 2700 y ≤648000

Z = 510x + 675 y

18. Max z = 50x + 60 y

$2x + y \leq 20$, $x + 2y \leq 12$, $x + 3y \leq 15$

$X \geq$, $y \geq 0$

(ii) Type A =6 , Type B =3

(iii) Max Profit Rs 480

19. 31011

20. (i) Linear Constraints

(ii) Linear

(iii) Bounded

(iv) Intersection

(v) Convex

21. (i) True

(ii) False

(iii) False

(iv) True

(v) True

BY PRASHANT TIWARI, PGT(MATHS) K V 2 INF LINES JAMNAGAR

CH-13

PROBABILITY

A (1-MARKS)

1) Twelve balls are distributed among three boxes. Find the probability that first box will contain three balls.
2) If each element of a second order determinant is either zero or one, what is the probability that the value the determinant is non-negative.
3) If A and B are two events such that P(A)=1/4, P(B)=1/2 and P(A∩B)=1/8, Find P(not A and not B)
4) A problem in mathematics is given to 3 students whose chances of solving it are ½, 1/3, ¼. What is the probability that the problem is solved?
5) If A and B are two independent events, Show that the probability of occurrence of at least one of A and B is given by 1- P(A')P(B').
6) If A and B are two independent events, find P(B) when P(A U B) = 0.60 and P(A) = 0.35.
7) Given P(A) = 0.3, P(B) = 0.2. find P(B/A) if A and B are mutually exclusive events.
8) A bag contains 5 brown and 4 white socks. A man pulls out 2 socks. Find the probability that these socks are of same colour.
9) Out of 30 consecutive integers, 2 are chosen at random. Find the probability that their sum is odd.
10) If P(A) = 7/13, P(B) = 9/13 and P(A∩B) = 4/13. Find P(A'/B).

B (2MARKS)

1) Two integers are selected at random from integers 1 to 11. If the sum is even, find the probability that both numbers are odd.

2) An urn contains 5 white and 8 black balls. Two successive drawing of three balls at a time are made such that the ball are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and second grow gives 3 black balls.

3) A committee of 4 Students is selected at random from a group consisting of 8 boys and 4 girls. Given that there is at least one girl in the committee, calculate the probability that there are exactly 2 girls in the committee.

4) A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected.

5) A bag contains 5 red marbles and 3 black marbles. Three Marbles are drawn one by one without replacement. What is the probability that at least one of the three Marbles drawn be black, if the first marble is red?

6) From a set of 100 cards numbered 1 to 100, one card is drawn at random. Find the probability that the number on the card is divisible by 6 or 8, but not by 24.

7) A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Find the probability of occurrence of A.

C (3 MARKS)

1) Consider the experiment of tossing a coin. If the coins shows head toss it again but if it shows tail then throw a die. Find the conditional probability of the event of 'the die shows a number greater than 4, given that there is at least one tail'.

2) For a loaded die, the probabilities of outcomes are given as under
P(1)=P(2)=2/10, P(3)=P(5)=1/10 and P(4)=1/10
the die is thrown two time. Let A and B be the events are as defined below

A=Getting same number each time, B=Getting a total score of 10 or more. Determine whether or not A and B are independent events.

3) A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to
   i) Contradict each other in stating the same fact?
   ii) Agree in stating the same fact ?
4) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts. Find the probability of the missing card to be a heart.
5) A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
6) A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that all balls are white ?

D (5 MARKS)

1) A can hit a target 4 times in 5 shots, B 3 times in 4 shots, and C 2 times in 3 shots. Calculate the probability that
   i) A,B,C all may hit
   ii) B, C may hit and A may not hit
   iii) Any two of A, B and C will hit the target
   iv) None of them will hit the target
2) A and B through alternatively a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. Find their respective chance of winning if A begins.
3) A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible what is the probability that the letter has come from
   i) TATANAGAR ii) CALCUTTA
4) And urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls in a random draw of three balls

5) Two biased dice are thrown together. For the first die P(6)=1/2, other is scores being equally likely while for the second die, P(1)=2/5 other scores are equally likely. find the probability distribution of 'the number of ones seen'.

6) A die is loaded in such a way that an even number is twice likely to occur as an odd number. If the die is tossed twice, find the probability distribution of the random variable X representing the perfect squares in the two tosses.

7) A bag I contains 5 red and 4 white balls and bag II contains 3 red and 3 white balls. Two balls are transferred from the bag I to the bag II and then one ball is drawn from the bag II. If the ball drawn from the bag II is red then find the probability that one red and one white ball are transferred from the bag I to the bag II.

8) A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, what is the probability that the missing card is black.

CASE STUDY

1) In a village there are Mohalla A B and C. In Mohalla A 300 farmers out of 500 believe in new technology of agriculture, In Mohalla B 280 farmers out of 400 believe in new technology of agriculture and in Mohalla C 480 farmers out of 600 believe in new technology of agricultur. A farmer is selected at random from village

i) The conditional probability that a farmer believe in new technology if he belongs to Mohalla A

(a) 7/10 (b) 4/5 (c) 3/5 (d) 1/5

ii) the probability that Mohalla A selected and selected farmer believe in new technology of agriculture

(a) 2/5 (b) 3/5 (c) 1/5 (d) none of these

iii) The conditional probability that a farmer believe in new technology if he belong Mohalla C

(a) 3/5 (b) 4/5 (c) 2/5 (d) 1/5

iv) The total probability that a farmer believe in new technology of agriculture

(a) 7/10 (b) 3/10 (c) 2/7 (d) 8/10

v) District agriculture Officer select a farmer at random in village and he found that selected farmer believe in new technology of agriculture the probability that the farmer belongs to Mohalla B is

(a) 1/3 (b) 2/3 (c) 3/5 (d) 3/7

ANSWERS : PROBABILITY

A (1 MARK)

1) $\frac{{}^{12}_{3}C \times 2^9}{3^{12}}$ 2) 13/16 4) 3/4 5) 3/8

6) 5/13 7) 0 8) 4/9 9) 15/29 10) 5/9

B (2 MARKS)

3/5 , 2)7/429 3) 168/425 4) 1/4 5)25/56 6)1/5 7) ½ or 1/3

C (3 MARKS)

1) 2/9 2) P(A).P(B)= P(A∩B)= 1/50 independent

3) i) 42/100, (ii) 58/100 4) 11/50 5) 3/8 6) 3/5

D (5 MARKS)

1 (i)2/5 (ii)1/10 (iii)13/10 (iv)1/60 2)P(A wins)=30/61, P(Bwins) =31/61 3) (i) 4/11 , (ii) 7/11

4. 5.

x	1	2	3	4
P(x)	1/2	1/4	1/8	1/8

X	0	1	2
P(x)	27/50	21/50	2/50

6

X	0	1	2
P(X)	4/9	4/9	1/9

7) 20/37   8) 2/3

CASE STUDY

(i) (c)3/5 (ii) (c) 1/5 (iii) (b)4/5 (iv) (a) 7/10 (v) (a) 1/3

SOLUTION

CASE STUDY

E_1= Selecting Mohalla A, E_2= Selecting Mohalla B, E_3= Selecting Mohalla C

A= Farmer believe in new technology of agriculture

$P(E_1)=1/3$, $P(E_2)=1/3$, $P(E_3)=1/3$

$P(A/E_1)=3/5$ $P(A/E_1)=7/10$ $P(A/E_1)=4/5$

(i) $P(A/E_1)=3/5$

(ii) By multiplication rule of probability

$P(E_1 \cap A)=P(E_1).P(A/E_1)=(1/3)\times(3/5)=1/5$

(iii) $P(A/E_3)=4/5$

(iv) By total probability theorm

$P(A)= P(E_1).P(A/E_1)+ P(E_2).P(A/E_2)+ P(E_3).P(A/E_3)$

$$=\frac{1}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{7}{10}+\frac{1}{3}\times\frac{4}{5}$$

$$=\frac{3}{15}+\frac{7}{30}+\frac{4}{15} = \frac{6+7+8}{30}=\frac{21}{30}=\frac{7}{10}$$

(v) By Baye's theorm

$$P(E_2/A)=\frac{P(E2).P(A/E2)}{P(E1).P(A/E1)+P(E2).P(A/E2)+P(E3).P(A/E3)}$$

$$=\frac{\frac{1}{3}\times\frac{7}{10}}{\frac{1}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{7}{10}+\frac{1}{3}\times\frac{4}{5}} = \frac{7}{21} = \frac{1}{3}$$

SOLUTION OF SOME PROBLEM:

SECTION A (1 Mark)

1) Since each ball can be put into any one of the three boxes. So, the total number of ways in which 12 balls can be put into three boxes is 3^{12}

out of 12 balls three balls can be chosen in ${}^{12}_{3}C$ ways. Put these 3 balls in the first box. Now remaining 9 balls are to be put in the remaining two boxes that can be done in 2^9ways.

so the total number of ways in 3 balls can be put in the first box and remaining 9 in other two boxes is ${}^{12}_{3}C \times 2^9$

Hence the required probability $\frac{{}^{12}_{3}C\times 2^9}{3^{12}}$

6) P(A U B) = P(A) + P(B) - P(A ∩ B)

Now since the two events are independent we get:

P(A ∩ B) = P(A) * P(B)

=> 0.6 = 0.35 + P(B) - 0.35*P(B)

=>0.25 = P(B) - 0.35*P(B)

=>0.25 = 0.65*P(B)

=> P(B) = 0.25/0.65 = 5/13

9) Total cases = ${}^{30}C_2$ = 435

The sum is odd, if one of the integers is odd and the other integer is even. So, the favourable cases = ${}^{15}C_1 . {}^{15}C_1$

(As, out of 30 consecutive integers, there will be 15 odd integers and 15 even integers.

Hence, the required probability = (15. 15)/435 =225/435 =15/29.

SECTION B (2 MARKS)

1) Out of integers from 1 to 11 there are 5 even integers and 6 odd integers.

consider the following events:

A= both the number chosen are odd,

B= the sum of the numbers chosen is even

P(B)= Probability getting the sum as an even number

$=\frac{{}^{5}_{2}C+{}^{6}_{2}C}{{}^{11}_{2}C}$ (either both are odd or both are evev)

P(A∩B)= Probability of selecting two odd number $=\frac{{}^{6}_{2}C}{{}^{11}_{2}C}$

required probability=Probability that the two number chosen are odd if it is given that the sum of the numbers chosen is even.

$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{{}^{6}_{2}C}{{}^{5}_{2}C+{}^{6}_{2}C}$ = 3/5

5) As the first drawn ball is red (R) and at least one of the remaining two

balls has to be black (B) , only following order of ball drawing is possible

RBB or RBR or RRB

So,

Probability =(5/8 x 3/7 x 2/6)+(5/8 x 3/7 x 4/6)+(5/8 x 4/7 x 3/6)

= 30/336 + 60/336 + 60/336

= 150/336

= 25/56

So,

Probability that at least one of the three marbles drawn will be black, if the first marble is red = 25/56

SECTION C (3 MARKS)

5) Let E_1 ,E_2 are the events that six occurs and six does not occur in throwing a die respectively.

$P(E_1) = \frac{1}{6}$,$P(E_2) = \frac{5}{6}$

Let A be the event that the man reports that six occurs on the die

$P(A/E_1) = \frac{3}{4}$, $P(A/E_3) = \frac{1}{4}$,

By Bay's theorem

$$P(E_1/A) = \frac{P(E_1)P(\frac{A}{E_1})}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)} = \frac{3}{8}$$

6) E_1= 2 white balls in bag, E_2= 3 white balls in bag,

E_3= 4 white balls in bag A= Drawn balls are white

$P(E_1)=1/3$, $P(E_2)=1/3$, $P(E_3)=1/3$

$P(A/E_1)=\frac{{}^2_2C}{{}^4_2C}=1/6$ $P(A/E_1)=\frac{{}^3_2C}{{}^4_2C}=3/6$ $P(A/E_1)=\frac{{}^4_2C}{{}^4_2C}=1$

By applying Baye's theorem

$$P(E_3/A)=\frac{P(E3).P(A/E3)}{P(E1).P(A/E1)+P(E2).P(A/E2)+P(E3).P(A/E3)}$$

$$=\frac{\frac{1}{3}\times 1}{(\frac{1}{3}\times\frac{1}{6})+(\frac{1}{3}\times\frac{3}{6})+(\frac{1}{3}\times 1)} = \frac{1}{\frac{1}{6}+\frac{3}{6}+1} = \frac{3}{5}$$

SECTION C (5 MARKS)

2) Let A be the event that A gets a total of 6 and B be the event of B getting total of 7

A : gets a total of 6,

A = {1,5), (5,1), (2,4), (4,2), (3,3)}

B : gets a total of 7,

B = {1,6), (6,1), (2,5), (5,2), (3,4), (4,3)}In a single throw,

P(A)=5/36 P(A')=31/36 P(B)=6/36 P(B')=30/36

Since A start the games

P(A wins)=P(A)+P(A'B'A)+P(A'B'A'B'A)+……….

=(5/36)+(31/36 x 30/36 x 5/36)+

+(31/36 x 30/36 x 31/36 x 30/36 x 5/36)+…….

$$=\frac{5/36}{1-\left(\frac{31}{36}\right)\left(\frac{30}{36}\right)} = \frac{30}{61} \qquad \left(S_\infty=\frac{a}{1-r}\right)$$

P(B wins)=1- P(A wins)=1-(30/61)=30/61

6) let p be the probability of getting on odd number in a single throw of a die. Then probability of getting an even numbers is 2p. We have,

P(1)+P(2)+ P(3)+P(4)+ P(5)+P(6)=1

p+2p+p+2p+p+2p+p=1

p=1/9

Now ,

Probability of getting a perfect square (1 or 4) in a single throw of a die

P(1)+P(4)=p+2p=3p=3×(1/9)=1/3

Since X denotes the number of perfect squares in two tosses. Then X = 0,1,2

P(0)=probability of not getting perfect squares in both the tosses= $\left(\frac{2}{3}\times\frac{2}{3}\right)=\frac{4}{9}$

P(1)= probability of getting perfect squares in one of the two the tosses = $\left(\frac{1}{3}\times\frac{2}{3}\right)+\left(\frac{2}{3}\times\frac{1}{3}\right)=\frac{4}{9}$

P(2)=probability of getting perfect squares in both the tosses= $\left(\frac{1}{3}\times\frac{1}{3}\right)=\frac{1}{9}$

Hence the probability distribution of X is

X	0	1	2
P(X)	4/9	4/9	1/9

BY MANISH KUMAR, PGT (MATHS) KV NO .1, SURAT